ESSENTIAL ICSE

MATHEMATICS

FOR CLASS 6

CONTENTS

1

Number Systems

Numbers are important in mathematics. They help us count concrete objects. We have studied about numbers in our previous classes. In this chapter, we shall review them and move forward.

Hindu–Arabic number system

A numeral is a symbolic representation of a number and a number system or numeral system is a system for writing numerals. The system that is now generally used is the Hindu–Arabic number system. It was developed by Hindu mathematicians and popularised by the Arabs.

In this system, the ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 are used to write a numeral. These symbols are called digits. 0, 2, 4, 6 and 8 are the even digits, while 1, 3, 5, 7 and 9 are the odd digits. Such a number system is called a denary system because counting is done in tens, or ten is the base of the system.

Examples (i) Ten 'units' make one 'ten': $10 \times 1 = 10$.

(ii) Ten 'tens' make one 'hundred': $10 \times 10 = 100$.

(iii) Ten 'hundreds' make one 'thousand': $10 \times 100 = 1000$.

Face value and place value of a digit

In the Hindu–Arabic system, the digits in a numeral have place values that depend on their position in the numeral. The face values, or actual values of the digits, however, do not change with their position. For example, in the numeral 235, the face value of the digit 2 is two, while its place value is two hundred.

Table 1.1 Place values in India (up to ten lakhs)

Lakhs		Thousands		Ones (units)		
Ten lakhs	Lakhs	Ten thousands	Thousands	Hundreds	Tens	Ones
7	5	4	3	6	1	9

The numeral shown in the table would be expressed in words as 'seventy-five lakh forty-three thousand six hundred and nineteen'. We group the places together into periods of lakhs, thousands and ones. Even when writing a number (or numeral) we group together the digits into periods by using commas or spaces. Thus, the number in Table 1.1 would be written as 75 43 619 or 75,43,619.

Number System Extended Further

The greatest 2-digit number is 99.

EXAMPLE **Compare the numbers.**

(i) 8 325 and 14 103 **(ii) 60 714 and 52 130**
(iii) 46 525 and 48 255 **(iv) 2 34 178 and 2 34 700**

Solution

(i) The first number has four digits while the second has five digits. So, the second number is bigger.

(ii) The two numbers have an equal number of digits. However, 6 > 5. So, 60 714 is the bigger number.

(iii) 46 525
48 255
↓└→8 > 6
=
∴ 48 255 > 46 525.

(iv) 2 34 178
2 34 700
↓↓↓ └→ 7 > 1
= = =
∴ 2 34 700 > 2 34 178.

Solved Examples

EXAMPLE 1 **Express the following numbers in words.**

(i) 8 40 36 251 **(ii) 3 15 79 36 480** **(iii) 7 55 30 060**

Solution We first write the numbers in an Indian place-value chart.

	Arabs	Crores		Lakhs		Thousands		Ones		
(i)			8	4	0	3	6	2	5	1
(ii)	3	1	5	7	9	3	6	4	8	0
(iii)			7	5	5	3	0	0	6	0

Then the numbers are the following.

(i) Eight crore forty lakh thirty-six thousand two hundred and fifty-one

(ii) Three arab fifteen crore seventy-nine lakh thirty-six thousand four hundred and eighty

(iii) Seven crore fifty-five lakh thirty thousand and sixty

EXAMPLE 2 **Write the face value and place value of the odd digits in the following.**
(i) 4 76 53 980 **(ii) 81 65 03 40 729**

Solution

(i)

Odd digit	*Face value*	*Place value*
7	Seven	70 00 000 (seventy lakh)
5	Five	50 000 (fifty thousand)
3	Three	3 000 (three thousand)
9	Nine	900 (nine hundred)

(ii)

Odd digit	*Face value*	*Place value*
1	One	1 00 00 00 000 (one arab)
5	Five	5 00 00 000 (five crore)
3	Three	3 00 000 (three lakh)
7	Seven	700 (seven hundred)
9	Nine	9 (nine)

EXAMPLE 3 **Write (i) 40 37 158 and (ii) 91 35 66 407 in the expanded form.**

Solution (i) $40\ 37\ 158 = 4 \times 1000000 + 0 \times 100000 + 3 \times 10000 + 7 \times 1000 + 1 \times 100 + 5 \times 10 + 8 \times 1.$

(ii) $91\ 35\ 66\ 407 = 9 \times 100000000 + 1 \times 10000000 + 3 \times 1000000 + 5 \times 100000 + 6 \times 10000 + 6 \times 1000 + 4 \times 100 + 0 \times 10 + 7 \times 1.$

EXAMPLE 4 **Write the greatest and the smallest numbers of eight digits that can be formed by using all the digits of the number 1 39 42 738.**

Solution The eight digits in the descending order are 9, 8, 7, 4, 3, 3, 2, 1.

∴ the greatest number of eight digits made by these is 9 87 43 321,

and the smallest number is 1 23 34 789.

EXAMPLE 5 **How many 3-digit numbers can you form by using the digits 9, 0, 5 only once?**

Solution Putting 0 at the units place, the possible numbers are 950 and 590.
Putting 0 at the tens place, the possible numbers are 905 and 509.
Putting 0 at the hundreds place would give us 2-digit numbers.
So, the required number of 3-digit numbers is 4.

EXAMPLE 6 **Rewrite each of the following numbers using an international place-value chart:**
(i) 71 235 (ii) 13 67 345 (iii) 2 17 56 849

Solution We arrange the given numbers in an international place-value chart.

	Millions			Thousands			Ones			
	HM	TM	M	HTh	TTh	Th	H	T	O	
(i)					7	1	2	3	5	71 235
(ii)			1	3	6	7	3	4	5	1 367 345
(iii)		2	1	7	5	6	8	4	9	21 756 849

In the international system, we write them in words as follows.

(i) Seventy-one thousand two hundred and thirty-five

(ii) One million three hundred sixty-seven thousand three hundred and forty-five

(iii) Twenty-one million seven hundred fifty-six thousand eight hundred and forty-nine.

EXAMPLE 7 **Arrange the following numbers in the ascending order:**
5123687, 21674116, 5132786, 24765829, 978678

Solution We arrange the given numbers in the following place-value chart.

Crores	Ten lakhs	Lakhs	Ten thousands	Thousands	Hundreds	Tens	Ones
	5	1	2	3	6	8	7
2	1	6	7	4	1	1	6
	5	1	3	2	7	8	6
2	4	7	6	5	8	2	9
		9	7	8	6	7	8

Out of the given numbers, two are 8-digit numbers, two are 7-digit numbers, and one is a 6-digit number. Obviously, the 6-digit number, i.e. 978678, is the smallest one.

For the 7-digit numbers, we have 5123687 < 5132786.

For the 8-digit numbers, we have 21674116 < 24765829.

So, 978678 < 5123687 < 5132786 < 21674116 < 24765829.

Hence, the given numbers in the ascending order are

978678, 5123687, 5132786, 21674116, 24765829.

EXAMPLE 8 **Arrange the following numbers in the descending order:**
74964502, 5478786, 74982401, 4789899, 638542

Solution We arrange the given numbers in the following place-value chart.

Crores	Ten lakhs	Lakhs	Ten thousands	Thousands	Hundreds	Tens	Ones
7	4	9	6	4	5	0	2
	5	4	7	8	7	8	6
7	4	9	8	2	4	0	1
	4	7	8	9	8	9	9
		6	3	8	5	4	2

Out of the given numbers, one is a 6-digit number, two are 7-digit numbers, and two are 8-digit numbers.

For 8-digit numbers, 74982401 > 74964502.

For 7-digit numbers, 5478786 > 4789899.

The 6-digit number, i.e. 638542, is the smallest.

So, 74982401 > 74964502 > 5478786 > 4789899 > 638542.

Hence, the given numbers in the descending order are

74982401, 74964502, 5478786, 4789899, 638542.

EXERCISE 1A

1. Express the following numbers in words.
   (i) 5 30 600 (ii) 27 48 166 (iii) 8 10 03 547
   (iv) 35 07 46 012 (v) 4 02 16 30 507 (vi) 6 00 90 005
2. Write the following numbers in figures.
   (i) Sixty lakh forty thousand and twelve
   (ii) Eleven lakh six thousand and seven
   (iii) Seven crore nine lakh three thousand and five hundred
   (iv) Five lakh five thousand and fifty-five
   (v) Seven arab eight crore twelve lakh and five hundred
   (vi) Nine arab seven crore seven thousand and nine
3. Write the face value and place value of the even digits in each of the following.
   (i) 4 32 65 781 (ii) 21 67 30 859 (iii) 4 05 16 38 157
4. Write the difference of the place values of the odd digits in each of the following.
   (i) 34 67 802 (ii) 7 43 66 280 (iii) 2 57 84 084

5. Write the following numbers in the expanded form.
   (i) 5 81 74 632 (ii) 10 86 92 354 (iii) 36 50 762

6. (a) Write the greatest number of six digits that can be formed using 2, 7, 4, 0, 2, 5.
   (b) Write the smallest number of seven digits that can be formed using 8, 3, 5, 1, 4, 3, 9.
   (c) Write the greatest and the smallest 8-digit numbers without repeating digits.
   (d) How many 3-digit numbers can you form by using each of the digits 7, 8, 1 only once?

7. Write all the numbers greater than 5000 that can be formed by the following digits. Do not repeat any digit in the same number.
   (i) 2, 3, 6, 4 (ii) 0, 4, 8, 1 (iii) 1, 2, 4, 7

8. Rewrite each of the following numbers using an international place-value chart. Also, write their name in the international system.
   (i) 645723 (ii) 16793 (iii) 74839825
   (iv) 46027137 (v) 20000148 (vi) 98300001

9. Write each of the following in figures arranging them in an international place-value chart.
   (i) Six million seven thousand and eight
   (ii) Forty-three million three hundred six thousand and seventy
   (iii) Sixty million two hundred eleven thousand and seventy-eight
   (iv) Thirty-six million four hundred sixty-one thousand one hundred and forty-nine
   (v) Seventy-eight million five hundred twenty thousand eight hundred and five
   (vi) Eight hundred forty-three thousand two hundred and thirty-four

10. Arrange the following numbers in the ascending order.
   (i) 204567, 1742306, 98342643, 78320678, 89342076
   (ii) 6243786, 31766114, 35752589, 6254611, 876543
   (iii) 12348765, 21116111, 1409786, 30008123, 7206435
   (iv) 62371018, 5176253, 62381219, 5347887, 236786, 62382005

11. Arrange the following numbers in the descending order.
   (i) 71723546, 717289, 56123408, 71230091, 385413
   (ii) 500045, 6013274, 6014368, 20032367, 6013678
   (iii) 260606, 2707077,28080888, 28006666, 2313133
   (iv) 589899, 45734, 6123888, 99995, 10007234, 10011051

ANSWERS

1. (i) Five lakh thirty thousand and six hundred
   (ii) Twenty-seven lakh forty-eight thousand one hundred and sixty-six
   (iii) Eight crore ten lakh three thousand five hundred and forty-seven
   (iv) Thirty-five crore seven lakh forty-six thousand and twelve
   (v) Four arab two crore sixteen lakh thirty thousand five hundred and seven
   (vi) Six crore ninety thousand and five
2. (i) 60 40 012 (ii) 11 06 007 (iii) 7 09 03 500 (iv) 5 05 055 (v) 7 08 12 00 500 (vi) 9 07 00 07 009
3. (i) For the digit 4: face value = 4, place value = 40000000 (four crore)
   For the digit 2: face value = 2, place value = 200000 (two lakh)
   For the digit 6: face value = 6, place value = 60000 (sixty thousand)
   For the digit 8: face value = 8, place value = 80 (eighty)

(ii) For the digit 2: face value = 2, place value = 200000000 (twenty crore)
For the digit 6: face value = 6, place value = 6000000 (sixty lakh)
For the digit 0: face value = 0, place value = 0
For the digit 8: face value = 8, place value = 800 (eight hundred)

(iii) For the digit 4: face value = 4, place value = 4000000000 (four arab)
For the digit 0: face value = 0, place value = 0
For the digit 6: face value = 6, place value = 600000 (six lakh)
For the digit 8: face value = 8, place value = 8000 (eight thousand)

4. (i) 29 93 000 (ii) 6 97 00 000 (iii) 43 00 000

5. (i) $5 \times 10000000 + 8 \times 1000000 + 1 \times 100000 + 7 \times 10000 + 4 \times 1000 + 6 \times 100 + 3 \times 10 + 2 \times 1$
(ii) $1 \times 100000000 + 0 \times 10000000 + 8 \times 1000000 + 6 \times 100000 + 9 \times 10000 + 2 \times 1000 + 3 \times 100$
$+ 5 \times 10 + 4 \times$
(iii) $3 \times 1000000 + 6 \times 100000 + 5 \times 10000 + 0 \times 1000 + 7 \times 100 + 6 \times 10 + 2 \times 1$

6. (a) 7 54 220 (b) 13 34 589 (c) 9 87 65 432 and 1 02 34 567 respectively (d) six

7. (i) 6 234, 6 243, 6 324, 6 342, 6 423, 6 432 (ii) 8 014, 8 041, 8 104, 8 140, 8 401, 8 410
(iii) 7 124, 7 142, 7 214, 7 241, 7 412, 7 421

8.

	Millions			Thousands			Ones			
	HM	TM	M	HTh	TTh	Th	H	T	O	
(i)				6	4	5	7	2	3	645 723
(ii)					1	6	7	9	3	16 793
(iii)		7	4	8	3	9	8	2	5	74 839 825
(iv)		4	6	0	2	7	1	3	7	46 027 137
(v)		2	0	0	0	0	1	4	8	20 000 148
(vi)		9	8	3	0	0	0	0	1	98 300 001

In words, these numbers are as follows.
(i) Six hundred forty-five thousand seven hundred and twenty-three
(ii) Sixteen thousand seven hundred and ninty-three
(iii) Seventy four million eight hundred thiry-nine thousand eight hundred and twenty-five
(iv) Forty-six million twenty-seven thousand one hundred and thirty-seven
(v) Twenty million one hundred and forty-eight
(vi) Ninety-eight million three hundred thousand and one

9.

	Millions			Thousands			Ones		
	HM	TM	M	HTh	TTh	Th	H	T	O
(i)			6	0	0	7	0	0	8
(ii)		4	3	3	0	6	0	7	0
(iii)		6	0	2	1	1	0	7	8
(iv)		3	6	4	6	1	1	4	9
(v)		7	8	5	2	0	8	0	5
(vi)				8	4	3	2	3	4

10. (i) 204567, 1742306, 78320678, 89342076, 98342643
(ii) 876543, 6243786, 6254611, 31766114, 35752589
(iii) 1409786, 7206435, 12348765, 21116111, 30008123
(iv) 236786, 5176253, 5347887, 62371018, 62381219, 62382005

11. (i) 71723546, 71230091, 56123408, 717289, 385413
(ii) 20032367, 6014368, 6013678, 6013274, 500045
(iii) 28080888, 28006666, 2707077, 2313133, 260606
(iv) 10011051, 10007234, 6123888, 589899, 99995, 45734

Operations on Large Numbers

We add, subtract, multiply and divide large numbers in the same way we do these operations on smaller numbers.

EXAMPLE 1 **Find the sum of 5308349, 47878598, 93864071 and 682573932.**

Solution

$$\begin{array}{r} 5308349 \\ +\ 47878598 \\ +\ 93864071 \\ +\ 682573932 \\ \hline 829624950 \end{array}$$

Hence, the required sum is 829624950.

EXAMPLE 2 **Subtract 2876092 from 37852061.**

Solution

$$\begin{array}{r} 37852061 \\ -\ 2876092 \\ \hline 34975969 \end{array}$$

So, 37852061 − 2876092 = 34975969.

EXAMPLE 3 **Simplify: 235647 − 4678896 − 783459 + 8058465.**

Solution We subtract the sum of the negative numbers from the sum of the positive numbers.

$$\begin{array}{r} 235647 \\ +\ 8058465 \\ \hline 8294112 \end{array} \qquad \begin{array}{r} 4678896 \\ +\ 783459 \\ \hline 5462355 \end{array}$$

$$\begin{array}{r} 8294112 \\ -\ 5462355 \\ \hline 2831757 \end{array}$$

Hence, the required answer is 2831757.

EXAMPLE 4 **Multiply 759283 by 1576.**

Solution

$$\begin{array}{r} 759283 \\ \times\ 1576 \\ \hline 4555698 \\ 5314981 \\ 3796415 \\ 759283 \\ \hline 1196630008 \end{array}$$

So, 759283 × 1576 = 1196630008.

EXAMPLE 5 **Divide 6408357 by 786.**

Solution

```
786)6408357(8153
   -6288
    1203
    -786
     4175
    -3930
     2457
    -2358
       99
```

Hence, quotient = 8153 and remainder = 99.

EXAMPLE 6 **Divide 25744733 by 4786.**

Solution

```
4786)25744733(5379
    -23930
      18147
     -14358
       37893
      -33502
        43913
       -43074
          839
```

Hence, quotient = 5379 and remainder = 839.

EXERCISE 1B

1. Find the following sums.
   (i) 7491856 + 56827562 + 40357381
   (ii) 82356712 + 59609 + 985743214 + 849325
   (iii) 39857253 + 871654 + 29578642 + 64320589
   (iv) 6853696 + 78640589 + 395442782 + 653424501
   (v) 845035621 + 89262505 + 4243456 + 321207 + 19905
2. (i) Subtract 894275 from 7184536.
   (ii) Subtract 5485362 from 30502071.
   (iii) Subtract 7854392 from 20202020.
   (iv) Subtract 12345618 from 31114507.
   (v) Subtract 87628793 from 215165821.
3. Simplify.
   (i) 23417692 – 176983523 – 82356 + 502135222
   (ii) 15307 – 6783059 + 98765402 – 47352735
   (iii) 573928 – 10395925 – 27830999 + 85423458
   (iv) 30200100 – 7900344 + 60708090 – 37285906
   (v) 25004937 – 8878999 – 628257 – 3125706

4. Multiply.
   (i) 586328 by 2387
   (ii) 347562 by 4268
   (iii) 798539 by 8905
   (iv) 2089567 by 9394
   (v) 7986053 by 51379

5. Divide.
   (i) 2376542 by 549
   (ii) 1760835 by 2763
   (iii) 72689368 by 8174
   (iv) 93205791 by 3854
   (v) 386286996 by 78963

ANSWERS

1. (i) 104676799 (ii) 1069008860 (iii) 134628138 (iv) 1134361568 (v) 938882694
2. (i) 6290261 (ii) 25016709 (iii) 12347628 (iv) 18768889 (v) 127537028
3. (i) 348487035 (ii) 44644915 (iii) 47770462 (iv) 45721940 (v) 12371975
4. (i) 1399564936 (ii) 1483394616 (iii) 7110989795 (iv) 19629392398 (v) 410315417087
5. (i) $Q = 4328$ and $R = 470$ (ii) $Q = 637$ and $R = 804$ (iii) $Q = 8892$ and $R = 6160$
(iv) $Q = 24184$ and $R = 655$ (v) $Q = 4892$ and $R = 0$

Solved Examples

EXAMPLE 1 **The population of a city was 23057893 in the year 2014. In the year 2015, it increased by 1078352. Find the population of the city in 2015.**

Solution Population of the city in the year 2015

= (population of the city in the year 2014) + (increase in population)

= 23057893 + 1078352 = 24136245.

$$\begin{array}{r} 23057893 \\ +\ 1078352 \\ \hline 24136245 \end{array}$$

Hence, the population of the city in the year 2015 was 24136245.

EXAMPLE 2 **The population of Mumbai is 12442373, that of Kolkata is 4496694 and that of Chennai is 4681087. What is the total population of these three cities?**

Solution Population of Mumbai = 12442373.

Population of Kolkata = 4496694.

Population of Chennai = 4681087.

∴ total population of these three cities = 12442373 + 4496694 + 4681087.

$$\begin{array}{r} 1\ 2\ 4\ 4\ 2\ 3\ 7\ 3 \\ +\ \ \ 4\ 4\ 9\ 6\ 6\ 9\ 4 \\ +\ \ \ 4\ 6\ 8\ 1\ 0\ 8\ 7 \\ \hline 2\ 1\ 6\ 2\ 0\ 1\ 5\ 4 \end{array}$$

Hence, the total population of the three cities is 21620154.

EXAMPLE 3 **The difference of two numbers is 87693076. If the smaller number is 49837835, find the bigger one.**

Solution Bigger number = difference + (smaller number)
= 87693076 + 49837835.

$$\begin{array}{r} 87693076 \\ +\ 49837835 \\ \hline 137530911 \end{array}$$

Hence, the bigger number is 137530911.

EXAMPLE 4 **The total population of a city is 16785101. If the number of males is 8987326, find the number of females in the city.**

Solution Number of females in the city = (total population) – (number of males)
= 16785101 – 8987326.

$$\begin{array}{r} 16785101 \\ -\ 8987326 \\ \hline 7797775 \end{array}$$

Hence, the number of females in the city is 7797775.

EXAMPLE 5 **The cost of a mobile phone is ₹16355. Find the cost of 643 such mobile phones.**

Solution ∵ the cost of 1 mobile phone is ₹16355,
∴ the cost of 643 such mobile phones is ₹16355 × 643.

$$\begin{array}{r} 16355 \\ \times\ 643 \\ \hline 49065 \\ 65420 \\ 98130 \\ \hline 10516265 \end{array}$$

Hence, the cost of 643 mobile phones is ₹10516265.

EXAMPLE 6 **The mass of an empty gas cylinder is 15 kg 310 g. Find the total mass of 23 such cylinders.**

Solution Mass of 1 empty gas cylinder = 15 kg 310 g.
∴ mass of 23 such cylinders = (15 kg 310 g) × 23.

kg	g
15	310
×	23
352 kg	130 g

Hence, the total mass of 23 cylinders is 352 kg 130 g.

EXAMPLE 7 **The length of cloth required for a shirt is 2 m 25 cm. How much length of cloth will be required for 18 such shirts?**

Solution The length of cloth required for 1 shirt = 2 m 25 cm.
∴ the length of cloth required for 18 such shirts = 18 × (2 m 25 cm).

2 m	25 cm
×	18
40 m	50 cm

Hence, the length of cloth required for 18 shirts is 40 m 50 cm.

EXAMPLE 8 **A car covers 675 km in 12 hours. Find the speed of the car.**

Solution The distance covered in 12 hours is 675 km.

So, the distance covered in 1 hour is 675 km ÷ 12.

```
12) 675 km (56 km
   - 60
     75
    - 72
      3 km
 = 3 × 1000 m
12) 3000 m (250 m
      24
      60
     - 60
      00
     - 00
       ×
```

Hence, the speed of the car is 56 km 250 m per hour.

EXAMPLE 9 **A vessel has 70 L of cold drink. In how many bottles, each of 1 L 250 mL capacity, can it be filled?**

Solution 70 L = 70000 mL and 1 L 250 mL = 1250 mL.

$\therefore$ number of bottles $= \frac{70000 \text{ mL}}{1250 \text{ mL}} = \frac{70000}{1250}$.

```
1250) 70000 (56
    - 6250
       7500
       7500
         ×
```

Hence, 56 bottles can be filled.

EXERCISE 1C

1. The number of tourists in India during the last four consecutive years was 80276457, 72968325, 65831211 and 93752346 respectively. Find the total number of tourists in all the four years.
2. In an election contested by three candidates, the successful candidate got 624783 votes, and the other two candidates secured 462385 and 387293 votes. The number of invalid votes was 6123 and the number of people who did not vote was 12389. Find the total number of voters registered.
3. The number of books sold by a bookstore in the year 2012 was 1657329. In the next year, the number of books sold increased by 782315. How many books were sold in the year 2013? How many books were sold during these two years?

4. A cement company produced 2643597 bags of cement in the year 2012. The number of bags produced increased by 127576 in the year 2013. In the next year, the number of bags produced was 276359 more than those produced in the preceding year. How many bags were produced during these three years? How many bags were produced collectively during the years 2012 and 2014?

5. A number exceeds 42973858 by 7899305. Find the number.

6. What must be added to 23497526 to get 612131510?

7. The total population of a city is 34927801. If the number of males is 18352017, find the number of females in the city.

8. Aftab needed ₹5632675 to buy a property. He had ₹2888279 as savings. He borrowed ₹1274631 from his friends and relatives. How much is he still short of?

9. A man had ₹11265278 with him. He gave ₹7532225 to his wife, ₹1227373 to his daughter, and rest to his son. How much money did his son receive?

10. A dairy had 725 L of milk with it. It sold 678 L 250 mL of milk. How much milk was left with it?

11. The price of a calculator is ₹1725. What is the price of 367 such calculators?

12. A factory produces 7835 pens per day. How many pens will it produce in 289 days?

13. The distance between the house of Aditya and his school is 1 km 625 m. Every day he walks both the ways. Find the total distance covered in six days.

14. To stitch a shirt, 2 m 35 cm length of cloth is required. How much length of cloth is needed to stitch 11 such shirts?

15. A car covers 874 km in 16 hours. Find the speed of the car.

16. A vessel has 5 litres and 250 mL of curd. In how many glasses—each of 50 ml capacity—can it be filled?

17. For making 13 shirts of the same size, 29 m 25 cm length of cloth is needed. How much length of cloth is required for each shirt?

18. A rope of length 15 m is divided into 12 pieces of equal lengths. Find the length of each piece.

19. A firm distributed 928 kg 125 g of food grains among 225 people affected by flood. How much food grains did a person get?

ANSWERS

1. 312828339	2. 1492973	3. 2439644; 4096973
4. 8462302; 5691129	5. 50873163	6. 588633984
7. 16575784	8. ₹1469765	9. ₹2505680
10. 46 L 750 mL	11. ₹633075	12. 2264315 pens
13. 19 km 500 m	14. 25 m 85 cm	15. 54 km 625 m per hour
16. 105	17. 2 m 25 cm	18. 1 m 25 cm
19. 4 kg 125 g		

Approximation

It is not always necessary to give the exact number. Suppose there were 71346 people watching a football match in a stadium. 71346 is closer to 71000 than it is to 72000. Hence, we may say that the number of people in the stadium was approximately (or about) 71000, a number which is easier to remember. This is called approximation or rounding off.

Symbolically, we write $71346 \approx 71000$. The symbol '$\approx$' means 'is approximately equal to'.

Here we have rounded off 71346 to the nearest thousand. Similarly, we can round off numbers to the nearest ten, hundred, thousand, ten thousand, lakh, million, and so on.

Estimation

Estimation is finding a number that is close enough to the right answer. It gives a rough idea of the answer.

So, 71000 is an estimate of the number of persons watching the match in the above example.

Thus, 71346 estimated to the nearest thousand is 71000.

Rules for approximating (or rounding off) whole numbers

1. If the digit on the right of the rounding place is greater than or equal to 5, add 1 to the digit at the rounding place. Or else, do not change the digit at the rounding place.
2. Replace all digits to the right of the rounding place by zeros.

Examples (i) The approximate value of 7843 correct to the nearest ten is 7840, as the digit to the right of the rounding place (i.e., on the units place) is 3 and $3 < 5$.

(ii) The approximate value of 26761 correct to the nearest hundred is 26800, as the digit on the tens place is 6 and $6 > 5$.

(iii) 178543 rounded off to the nearest thousand is 179000, as the digit on the hundreds place is 5.

(iv) 764735 estimated to the nearest ten thousand is 760000, as the digit to the right of the rounding place is 4 and $4 < 5$.

Estimation of the sum or difference of two numbers

Rule Round off each number to the desired place and find the sum or difference.

EXAMPLE **Estimate the sum (58 + 73) to the nearest ten.**

Solution 58 estimated to the nearest ten is 60,
and 73 estimated to the nearest ten is 70.
Hence, the estimated sum is $60 + 70 = 130$.

EXAMPLE **Estimate the sum (456 + 242) to the nearest ten.**

Solution 456 estimated to the nearest ten is 460,
and 242 estimated to the nearest ten is 240.
So, the required estimation is 460 + 240 = 700.

EXAMPLE **Estimate the difference (742 – 379) to the nearest ten.**

Solution 742 estimated to the nearest ten is 740,
and 379 estimated to the nearest ten is 380.
So, the estimated difference is 740 – 380 = 360.

Estimation of products and quotients

Rule Round off each number to its greatest place. Multiply the rounded-off numbers to get the product.
Similarly, we can find the quotient.

EXAMPLE **Estimate the product of 34 and 77.**

Solution 34 is a two-digit number. So, we round off 34 to its greatest place, i.e., to the nearest ten.
34 rounded off to the nearest ten is 30.
Similarly, 77 rounded off to the nearest ten is 80.
Therefore, the estimated product is $30 \times 80 = 2400$.

EXAMPLE **Find the estimated quotient for $79 \div 21$.**

Solution 79 rounded off to the nearest ten is 80,
and 21 rounded off to the nearest ten is 20.
Here the estimated quotient is $80 \div 20 = 4$.

Solved Examples

EXAMPLE 1 **Round off the following as instructed.**
(i) 738469 to the nearest ten thousand
(ii) 1457723 to the nearest lakh
(iii) 89481477 to the nearest ten lakh
(iv) 213685926 to the nearest crore

Solution
(i) The digit to the right of the rounding place is 8, and 8 > 5. So, 738469 rounded off to the nearest ten thousand is 740000.
(ii) The digit to the right of the rounding place is 5. So, 1457723 rounded off to the nearest lakh is 1500000.
(iii) The digit to the right of the ten lakhs place is 4, and 4 < 5. So, 89481477 rounded off to the nearest ten lakh is 89000000.
(iv) The digit to the right of the crores place is 3, and 3 < 5. Hence, 213685926 rounded off to the nearest crore is 210000000.

EXAMPLE 2 **Estimate the sum (734 + 575) to the nearest hundred.**

Solution 734 estimated to the nearest hundred is 700,

and 575 estimated to the nearest hundred is 600.
Hence, the estimated sum is 700 + 600 = 1300.

EXAMPLE 3 **Estimate the sum (4723 + 24349 + 46750) to the nearest thousand.**

Solution 4723 estimated to the nearest thousand is 5000,
24349 estimated to the nearest thousand is 24000, and
46750 estimated to the nearest thousand is 47000.
Hence, the estimated sum is 5000 + 24000 + 47000 = 76000.

EXAMPLE 4 **Estimate the difference (6725 - 484) to the nearest hundred.**

Solution 6725 estimated to the nearest hundred is 6700,
and 484 estimated to the nearest hundred is 500.
Hence, the estimated difference is 6700 - 500 = 6200.

EXAMPLE 5 **Estimate the product of 77 and 383.**

Solution 77 rounded off to the greatest place, i.e., to the nearest ten, is 80.
383 is a three-digit number. So, we round off 383 to the nearest hundred.
383 rounded off to the nearest hundred is 400.
The estimated product is $80 \times 400 = 32000$.

EXAMPLE 6 **Find the estimated quotient for**
(i) 58 ÷ 34 **(ii) 316 ÷ 27** **(iii) 721 ÷ 32.**

Solution
(i) $58 \approx 60$ and $34 \approx 30$.
$\therefore\ 58 \div 34 \approx 60 \div 30 = 2$.
So, the estimated quotient is 2.
(ii) $316 \approx 300$ and $27 \approx 30$.
$\therefore\ 316 \div 27 \approx 300 \div 30 = 10$.
So, the estimated quotient is 10.
(iii) $721 \approx 700$ and $32 \approx 30$.
$\therefore$ $721 \div 32$ is aproximately equal to $700 \div 30$,
i.e. approximately equal to $70 \div 3$,
i.e. approximately equal to 23.
So, the estimated quotient is 23.

EXERCISE 1D

1. Find the approximate value of each of the following.
   (i) 573 correct to the nearest ten
   (ii) 3872 correct to the nearest hundred
   (iii) 24576 correct to the nearest thousand
   (iv) 1699999 correct to the nearest ten thousand
2. Round off the following as instructed.
   (i) 3168303 to the nearest lakh

(ii) 47235839 to the nearest ten lakh
(iii) 26948035 to the nearest crore

3. Approximate 68443675:
   (i) correct to the nearest ten.
   (ii) correct to the nearest hundred.
   (iii) correct to the nearest thousand.
   (iv) correct to the nearest ten thousand.
   (v) correct to the nearest lakh.
   (vi) correct to the nearest ten lakh.
   (vii) correct to the nearest crore.
4. Estimate each sum to the nearest ten.
   (i) 43 + 28 (ii) 13 + 58 (iii) 96 + 47
   (iv) 173 + 387 (v) 634 + 478 (vi) 251 + 732
5. Estimate each sum to the nearest hundred.
   (i) 437 + 322 (ii) 180 + 276 (iii) 789 + 235
   (iv) 2470 + 5492 (v) 3120 + 6370 (vi) 21301 + 18582
6. Estimate each sum to the nearest thousand.
   (i) 17478 + 31845 (ii) 12385 + 47812 (iii) 23659 + 58145
   (iv) 64580 + 21732
7. Estimate each difference to the nearest ten.
   (i) 96 – 27 (ii) 64 – 28 (iii) 508 – 237 (iv) 731 – 469
8. Estimate each difference to the nearest hundred.
   (i) 579 – 214 (ii) 858 – 477 (iii) 4613 – 2089 (iv) 6307 – 3428
9. Estimate each difference to the nearest thousand.
   (i) 46765 – 18785 (ii) 76964 – 28384 (iii) 51213 – 34370 (iv) 63425 – 41905
10. Estimate each of the following product.
   (i) 27×73 (ii) 43×58 (iii) 68×471
   (iv) 523×77 (v) 594×248 (vi) 674×389
11. Find the estimated quotient for each of the following.
   (i) $87 \div 27$ (ii) $98 \div 32$ (iii) $74 \div 34$
   (iv) $167 \div 36$ (v) $731 \div 22$ (vi) $815 \div 33$

ANSWERS

1. (i) 570 (ii) 3900 (iii) 25000 (iv) 1700000 **2.** (i) 3200000 (ii) 47000000 (iii) 30000000
3. (i) 68443680 (ii) 68443700 (iii) 68444000 (iv) 68440000 (v) 68400000 (vi) 68000000 (vii) 70000000
4. (i) 70 (ii) 70 (iii) 150 (iv) 560 (v) 1110 (vi) 980
5. (i) 700 (ii) 500 (iii) 1000 (iv) 8000 (v) 9500 (vi) 39900
6. (i) 49000 (ii) 60000 (iii) 82000 (iv) 87000 **7.** (i) 70 (ii) 30 (iii) 270 (iv) 260
8. (i) 400 (ii) 400 (iii) 2500 (iv) 2900 **9.** (i) 28000 (ii) 49000 (iii) 17000 (iv) 21000
10. (i) 2100 (ii) 2400 (iii) 35000 (iv) 40000 (v) 120000 (vi) 280000
11. (i) 3 (ii) 3 (iii) 2 (iv) 5 (v) 35 (vi) 26

2

Factors and Multiples

Factors

If two or more numbers are multiplied together to get a product then each of the numbers is called a factor (or divisor) of the product.

Examples (i) $5 \times 7 = 35$. So, 5 and 7 are factors of 35.

(ii) $3 \times 4 \times 6 = 72$. So, 3, 4 and 6 are factors of 72.

$1 \times$ (a number) is equal to the number itself. So,

1. 1 is a factor of every number, and
2. every number is a factor of itself.

EXAMPLE **Verify whether (i) 21 is a factor of 315 and (ii) 34 is a factor of 852.**

Solution (i)

$$\begin{array}{r} 15 \\ 21\overline{)315} \\ -21 \\ \hline 105 \\ -105 \\ \hline \times \end{array}$$

$\therefore$ $315 = 21 \times 15$.

$\therefore$ 21 is a factor of 315.

(ii)

$$\begin{array}{r} 25 \\ 34\overline{)852} \\ -68 \\ \hline 172 \\ -170 \\ \hline 2 \end{array}$$

$\therefore$ 34 does not divide 852 exactly.

$\therefore$ 34 is not a factor of 852.

Multiples

The product of two or more numbers is called a multiple of each of the numbers.

Examples (i) $5 \times 7 = 35$. So, 35 is a multiple of 5 as well as 7.

(ii) $2 \times 3 = 6$. So, 6 is a multiple of 2 as well as 3.

Characteristics of multiples

1. Every number is a multiple of itself, because
   $1 \times$ (a number) = the number itself.
2. Every number has a countless number of multiples, because every number can be multiplied by countless natural numbers.
   For example, the multiples of 2 are
   $2 \times 1, 2 \times 2, 2 \times 3, 2 \times 4, \ldots$.

A number that divides a bigger number without leaving a remainder is called a factor (or divisor) of the bigger number, while the bigger number is called its multiple.

Classification of numbers

Whole numbers are classified on the basis of their factors and multiples.

Even number

A number is called an even number if it is a multiple of 2. In other words, 2 is a factor of every even number, or every even number can be divided by 2.

The number 0 and the digits that are multiples of 2 are called even digits. So, 0, 2, 4, 6 and 8 are the five even digits.

Odd number

A number is called an odd number if it is not a multiple of 2. In other words, 2 is not a factor of any odd number, or no odd number can be divided by 2.

The five odd digits are 1, 3, 5, 7, 9.

> Any whole number is either odd or even. An even number must have an even digit in the ones place. An odd number has an odd digit in the ones place.

Prime number

A natural number other than 1 is called a prime number if it does not have a factor other than 1 and the number itself. Some prime numbers are 2, 3, 5, 7, 11, 13, 17 and 19.

> 2 is the only even prime number.

A Greek astronomer and mathematician named Eratosthenes came up with the following method for finding the prime numbers between 1 and 100. The method is referred to as the sieve of Eratosthenes.

Steps 1. Arrange the numbers in rows of 10.

2. The smallest prime number is 2. Encircle 2 and cross out 1 and all other even numbers.
3. The prime number next to 2 is 3. Encircle 3 and cross out all other multiples of 3. Many of these would have been crossed out already.
4. The next prime number after 3 is 5. Encircle 5 and cross out all the other multiples of 5 which have not been crossed out until now.
5. The prime number next to 5 is 7. Encircle 7 and cross out all the other multiples of 7 which have not been crossed out until now. Encircle the remaining numbers.

~~1~~	(2)	(3)	~~4~~	(5)	~~6~~	(7)	~~8~~	~~9~~	~~10~~
(11)	~~12~~	(13)	~~14~~	~~15~~	~~16~~	(17)	~~18~~	(19)	~~20~~
~~21~~	~~22~~	(23)	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	(29)	~~30~~
(31)	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	(37)	~~38~~	~~39~~	~~40~~
(41)	~~42~~	(43)	~~44~~	~~45~~	~~46~~	(47)	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	(53)	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	(59)	~~60~~

(61)	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	(67)	~~68~~	~~69~~	~~70~~
(71)	~~72~~	(73)	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	(79)	~~80~~
~~81~~	~~82~~	(83)	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	(89)	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	(97)	~~98~~	~~99~~	~~100~~

You will see that the encircled numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97. These are the all twenty-five prime numbers between 1 and 100.

Coprime numbers

Two natural numbers are called coprime numbers if they do not have a common factor other than 1. For example, 4 and 9; 5 and 11; and 3 and 8 are coprime numbers.

Composite number

A natural number is called a composite number if it has at least one factor other than 1 and the number itself. Some of the composite numbers are 4, 6, 8, 9, 10, 12, 14, 15, 16.

> All even numbers other than 2 are composite numbers. 1 is neither a prime number nor a composite number.

Perfect number

A number is called a perfect number if the sum of all its factors except itself is equal to the number.

For 6, the factors are 1, 2, 3 except 6. Also, $1 + 2 + 3 = 6$. So, 6 is a perfect number. Similarly, 28 is also a perfect number, since $28 = 1 + 2 + 4 + 7 + 14$.

Remember These

1. A number that divides a bigger number exactly is a factor of the bigger number, while the bigger number is a multiple of the first number.
2. The multiples of a number are obtained by multiplying it by 1, 2, 3, etc.
3. A number is called a prime number if it does not have a factor other than 1 and itself.
4. Two numbers are called coprime numbers if they do not have a common factor other than 1.
5. If a number has at least one factor other than 1 and itself, it is called a composite number.

EXERCISE 2A

1. (i) Is 35 a factor of 455? (ii) Is 13 a factor of 816?
   (iii) Are 5 and 3 factors of 1647? (iv) Are 2, 3 and 7 factors of 154?
   (v) Are 2, 3, 5, 7 and 11 factors of 4620?

2. (i) Write the first eight multiples of 7. (ii) Write the first ten multiples of 5.
(iii) Write the first six multiples of 11.
3. Write all the prime numbers between 1 and 50.
4. Make pairs of coprime numbers from 15, 21, 28, 16, 11.

ANSWERS

1. (i) Yes (ii) No (iii) 3 is a factor (iv) 2 and 7 are factors (v) Yes
2. (i) 7, 14, 21, 28, 35, 42, 49, 56 (ii) 5, 10, 15, 20, 25, 30, 35, 40, 45, 50 (iii) 11, 22, 33, 44, 55, 66
3. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47
4. 15, 28; 15, 16; 15, 11; 21, 16; 21, 11; 28, 11; 16, 11

Rules for Tests of Divisibility by 2, 3, 4, 5, 6, 7, 8, 9, 10 and 11

I. Test of divisibility by 2

A number is divisible by 2 if its ones digit is 0, 2, 4, 6 or 8.

Examples 1. Each of the numbers 10, 22, 54, 116, 3408 is divisible by 2.

2. The numbers 21, 43, 165, 847, 1049 are not divisible by 2.

II. Test of divisibility by 3

A number is divisible by 3 if the sum of the digits of the number is divisible by 3.

Examples 1. Consider the number 2565. Sum of its digits = $2 + 5 + 6 + 5 = 18$. 18 is divisible by 3. So, 2565 is divisible by 3.

2. Consider the number 91639.
Sum of its digits = $9 + 1 + 6 + 3 + 9 = 28$.
28 is not divisible by 3. So, 91639 is not divisible by 3.

III. Test of divisibility by 4

A number is divisible by 4 if the number formed by its last two digits is divisible by 4.

Examples 1. Consider the number 37964. The number formed by its last two digits is 64, which is divisible by 4. So, 37964 is divisible by 4.

2. Consider the number 241682. The number formed by its last two digits is 82, and 82 is not divisible by 4. So, 241682 is not divisible by 4.

IV. Test of divisibility by 5

Any number that ends in either 0 or 5 is divisible by 5.

Examples 1. Each of the numbers 70, 135, 5795, 783590 is divisible by 5.

2. None of the numbers 51, 753, 5057, 130559 is divisible by 5.

V. Test of divisibility by 6

A number is divisible by 6 if it is divisible by 2 and 3 both.

Examples 1. Consider the number 132. Its ones digit is 2. So, it is divisible by 2. Sum of its digits $= 1 + 3 + 2 = 6$, and 6 is divisible by 3. So, the number is divisible by 3 also. Therefore, it is divisible by 6.

2. None of the numbers 56, 133, 656, 7534 is divisible by 6.

VI. Test of divisibility by 7

A number is divisible by 7 if the difference between twice the ones digit and the number formed by the other digits is divisible by 7.

Examples 1. Consider the number 294. Here $29 - 4 \times 2 = 21$, and 21 is divisible by 7. So, 294 is divisble by 7.

2. Consider the number 3467. Here $346 - 7 \times 2 = 332$. Is 332 divisible by 7?
$33 - 2 \times 2 = 29$, and 29 is not divisible by 7. So, 332 and finally 3467 are not divisible by 7.

VII. Test of divisibility by 8

A number is divisible by 8 if the number formed by its last three digits is divisible by 8.

Examples 1. Consider the number 7528. The number formed by its last three digits is 528, which is divisible by 8. So, 7528 is divisible by 8.

2. Consider the number 36706. The number formed by its last three digits is 706, and 706 is not divisible by 8. So, 36706 is not divisible by 8.

3. The number 5000 is divisible by 8, as the number formed by its last three digits is 000, and 000 is divisible by 8.

VIII. Test of divisibility by 9

A number is divisible by 9 if the sum of its digit is divisible by 9.

Examples 1. Consider the number 370314.
Sum of its digits $= 3 + 7 + 0 + 3 + 1 + 4 = 18$, and 18 is divisible by 9. So, 370314 is divisible by 9.

2. Consider the number 1835927. Sum of its digits $= 1 + 8 + 3 + 5 + 9 + 2 + 7 = 35$. Since 35 is not divisible by 9, the number 1835927 is not divisible by 9.

IX. Test of divisibility by 10

A number is divisible by 10 if its ones digit is 0.

Examples 1. Each of the numbers 10, 750, 2560, 9000 is divisible by 10.

2. None of the numbers 11, 23, 35, 105, 4003 is divisible by 10.

X. Test of divisibility by 11

A number is divisible by 11 if the difference of the sum of its digits in the odd places and the sum of its digits in the even places is either 0 or divisible by 11.

Examples 1. Consider the number 79453. Sum of its digits in the odd places $= 3+4+7 = 14$. Sum of its digits in the even places $= 5+9 = 14$. Their difference $= 14-14 = 0$. So, 79453 is divisible by 11.

2. Consider the number 938278. Sum of its digits in the odd places $= 8+2+3 = 13$. Sum of its digits in the even places $= 7+8+9 = 24$. Their difference $= 24-13 = 11$. So, 938278 is divisible by 11.

3. Consider the number 1234567. Sum of its digits in the odd places $= 7+5+3+1 = 16$. Sum of its digits in the even places $= 6+4+2 = 12$. Their difference $= 16-12 = 4$, which is not divisible by 11. So, 1234567 is not divisible by 11.

Some Properties of Divisibility

PROPERTY 1 If a number is divisible by another number, it is divisible by each of the factors of that number.

Examples 1. 24 is divisible by 12. The factors of 12 are 1, 2, 3, 4, 6, 12. So, all the factors 1, 2, 3, 4, 6 and 12 divide 24.

2. Every number divisible by 9 is divisible by 3.

3. Every number divisible by 8 is divisible by 4.

PROPERTY 2 If a number is divisible by two coprime numbers then it is divisible by their product also.

Examples 1. The number 60 is divisible by 4 and 5. Also, 4 and 5 are coprimes. So, 60 is also divisible by $4 \times 5 = 20$.

2. 132 is divisible by both 4 and 6. But 4 and 6 are not coprimes. So, 132 is not divisible by $4 \times 6 = 24$.

3. Two prime numbers are always coprime. So, a number divisible by two prime numbers is also divisible by their product.

PROPERTY 3 If two given numbers are divisible by a number, their sum is also divisible by that number.

Examples 1. 24 and 32 are divisible by 4. Their sum $= 24+32 = 56$. So, 56 is also divisible by 4.

2. 35 and 28 are divisible by 7. Their sum, i.e. $35+28 = 63$, is also divisible by 7.

PROPERTY 4 If two given numbers are divisible by a number then their difference is also divisible by that number.

Examples 1. 24 and 9 are divisible by 3. $24-9 = 15$, which is also divisible by 3.

2. 70 and 42 are divisible by 14. Their difference $= 70-42 = 28$. Clearly, 28 is also divisible by 14.

EXERCISE 2B

1. Test the divisibility of the following numbers by 2.
   (i) 37641 (ii) 7950 (iii) 476134 (iv) 979052
   (v) 1379358

2. Test the divisibility of the following numbers by 3.
   (i) 579 (ii) 605723 (iii) 4738923 (iv) 8097435
   (v) 9053

3. Test the divisibility of the following numbers by 4.
   (i) 712 (ii) 3538 (iii) 79006 (iv) 137056
   (v) 897640

4. Test the divisibility of the following numbers by 5.
   (i) 5509 (ii) 13470 (iii) 98501 (iv) 73825
   (v) 49350

5. Test the divisibility of the following numbers by 6.
   (i) 822 (ii) 3090 (iii) 911356 (iv) 461754
   (v) 76745

6. Test the divisibility of the following numbers by 7.
   (i) 25347 (ii) 54067 (iii) 28826 (iv) 2485
   (v) 854

7. Test the divisibility of the following numbers by 8.
   (i) 3864 (ii) 8116 (iii) 56688 (iv) 131992
   (v) 7305120

8. Test the divisibility of the following numbers by 9.
   (i) 4266 (ii) 65319 (iii) 30465 (iv) 413973
   (v) 859671

9. Test the divisibility of the following numbers by 10.
   (i) 2130 (ii) 500056 (iii) 79000 (iv) 83055
   (v) 555

10. Test the divisibility of the following numbers by 11.
   (i) 3773 (ii) 2230908 (iii) 324368 (iv) 82819
   (v) 1111111

11. In each of the following numbers, replace * by the smallest digit to make it divisible by 3.
   (i) 15*2 (ii) 7*5631 (iii) 432*71 (iv) 1502*
   (v) 738592*8

12. In each of the following numbers, replace * by the smallest digit to make it divisible by 9.
   (i) 35*7 (ii) 4*799 (iii) 712*35 (iv) 973640*5
   (v) 70*543

13. In each of the following numbers, replace • by the smallest digit to make it divisible by 11.

(i) 16•43 (ii) 230•75 (iii) 16942•3 (iv) 74•8359

(v) 810291•3

14. Give an example of a number

(i) which is divisible by 2 but not by 4.

(ii) which is divisible by 3 but not by 6.

(iii) which is divisible by 4 but not by 8.

(iv) which is divisible by both 2 and 8 but not by 16.

(v) which is divisible by both 3 and 6 but not by 18.

(vi) which is divisible by both 4 and 8 but not by 32.

ANSWERS

1. (ii), (iii), (iv), (v) are divisible by 2.
2. (i), (iii), (iv) are divisible by 3.
3. (i), (iv), (v) are divisible by 4.
4. (ii), (iv), (v) are divisible by 5.
5. (i), (ii), (iv) are divisible by 6.
6. (i), (iii), (iv), (v) are divisible by 7.
7. (i), (iii), (iv), (v) are divisible by 8.
8. (i), (iii), (iv), (v) are divisible by 9.
9. (i), (iii) are divisible by 10.
10. (i), (iii), (iv) are divisible by 11.
11. (i) 1 (ii) 2 (iii) 1 (iv) 1 (v) 0
12. (i) 3 (ii) 7 (iii) 0 (iv) 2 (v) 8
13. (i) 6 (ii) 1 (iii) 5 (iv) 9 (v) 1
14. (i) 14 (ii) 9 (iii) 12 (iv) 24 (v) 24 (vi) 48

Factorisation and Prime Factorisation

Writing a number as the product of two or more numbers is called factorisation of the number. When a number is written as a product of prime numbers only, the process is called prime factorisation.

Examples (i) $120 = 4 \times 6 \times 5$ is a factorisation of 120, but not the prime factorisation.

(ii) $120 = 2 \times 2 \times 2 \times 3 \times 5$ is the prime factorisation of 120.

Method for prime factorisation

Remember the following rules about division by prime numbers.

1. Every even number is divisible by the prime number 2.
2. A number that has 0 or 5 in the units place is divisible by 5.
3. A number is divisible by 3 if the sum of its digits is divisible by 3. For example, 1731 is divisible by 3 because $1+7+3+1=12$ is divisible by 3.
4. A number is divisible by 11 if the sum of the digits in the even places is equal to the sum of the digits in the odd places or the difference of the two sums is divisible by 11. Example, 3476 is divisible by 11 as the sum of the digits in the even places is $7 + 3 = 10$ and the sum of the digits in the odd places is $6 + 4 = 10$, and these two sums are equal.

EXAMPLE **Express 162 as a product of prime numbers.**

Solution

$162 = 2 \times 81$ ($\because$ 162 is even, 2 is a factor)

$= 2 \times 3 \times 27$ (in 81, sum of digits $= 8 + 1 = 9$, which is divisible by 3)

$= 2 \times 3 \times 3 \times 9$

$= 2 \times 3 \times 3 \times 3 \times 3.$

Alternative method

$$\begin{array}{r|l} 2 & 162 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$\therefore$ $162 = 2 \times 3 \times 3 \times 3 \times 3.$

EXAMPLE **Carry out the prime factorisation of 8250.**

Solution

$8250 = 2 \times 4125$ ($\because$ 8250 is even, 2 is a factor)

$= 2 \times 5 \times 825$ (in 4125, the digit in the ones place is 5)

$= 2 \times 5 \times 5 \times 165$

$= 2 \times 5 \times 5 \times 5 \times 33 = 2 \times 5 \times 5 \times 5 \times 3 \times 11.$

EXAMPLE **Express 1197 as a product of primes.**

Solution

$$\begin{array}{r|l} 3 & 1197 \\ \hline 3 & 399 \\ \hline 7 & 133 \\ \hline 19 & 19 \\ \hline & 1 \end{array}$$

(the sum of digits is $1 + 1 + 9 + 7 = 18$, which is divisible by 3)

($\because$ 133 is not divisible by 2, 5, 3 or 11, we check whether 7 is a factor)

(19 is a prime)

$\therefore$ $1197 = 3 \times 3 \times 7 \times 19.$

From the preceding examples, we get the following method for prime factorisation.

Steps

1. Take out the factor 2 if the number is even. Continue this step until you get an odd digit in the ones place.
2. Take out the factor 5 if the number you get after Step 1 has 5 in the ones place. Continue this step until you get a number without 5 in the ones place.
3. Take out the factor 3 if the sum of the digits of the number after Step 2 is a multiple of 3 (that is, divisible by 3). Continue this step till you get a number that does not have 3 as a factor.
4. Take out the factor 11 if the sum of the digits in the even places is equal to the sum of the digits in the odd places or the difference of the two sums is divisible by 11. Repeat the step if required.
5. Try (by division) to take out prime factors like 7, 13 and 17 until 1 is left as the dividend.

EXERCISE 2C

Express each of the following as a product of prime numbers.

1. 36	**2.** 42	**3.** 72	**4.** 240
5. 315	**6.** 693	**7.** 1680	**8.** 1232
9. 2310	**10.** 1764	**11.** 23595	**12.** 24640

ANSWERS

1. $2 \times 2 \times 3 \times 3$	**2.** $2 \times 3 \times 7$	**3.** $2 \times 2 \times 2 \times 3 \times 3$
4. $2 \times 2 \times 2 \times 2 \times 3 \times 5$	**5.** $3 \times 3 \times 5 \times 7$	**6.** $3 \times 3 \times 7 \times 11$
7. $2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 7$	**8.** $2 \times 2 \times 2 \times 2 \times 7 \times 11$	**9.** $2 \times 3 \times 5 \times 7 \times 11$
10. $2 \times 2 \times 3 \times 3 \times 7 \times 7$	**11.** $3 \times 5 \times 11 \times 11 \times 13$	**12.** $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 7 \times 11$

Highest Common Factor (HCF)

Let us consider the numbers 18 and 24.

$18 = 2 \times 3 \times 3$; so 1, 2, 3, 6, 9 and 18 are factors of 18.

$24 = 2 \times 2 \times 2 \times 3$; so 1, 2, 3, 4, 6, 8, 12 and 24 are factors of 24.

Clearly, the factors that the two numbers have in common are 1, 2, 3 and 6. Of these, 6 is the largest, so it is called the highest common factor of the two numbers.

The highest common factor (HCF) of two or more numbers is the largest number which divides each of the numbers. The HCF of some numbers is also known as their greatest common divisor (GCD).

The HCF of two or more numbers can be found by (a) prime factorisation and (b) division.

To find the HCF by prime factorisation

Steps
1. Express each number as a product of prime numbers.
2. Find the factors common to all the numbers.
3. Multiply the common factors. The product is the required HCF.

EXAMPLE **Find the HCF of 84 and 48.**

Solution On prime factorisation, we have

$84 = 2 \times 42 = 2 \times 2 \times 21 = 2 \times 2 \times 3 \times 7.$

$48 = 2 \times 24 = 2 \times 2 \times 12 = 2 \times 2 \times 2 \times 6 = 2 \times 2 \times 2 \times 2 \times 3.$

The common factors are 2, 2 and 3.

$\therefore$ HCF $= 2 \times 2 \times 3 = 12.$

EXAMPLE **Find the HCF of 216, 315 and 180.**

Solution On prime factorisation, we have

$216 = 2 \times 108 = 2 \times 2 \times 54$

$= 2 \times 2 \times 2 \times 27 = 2 \times 2 \times 2 \times 3 \times 9 = 2 \times 2 \times 2 \times 3 \times 3 \times 3,$

$315 = 3 \times 105 = 3 \times 3 \times 35 = 3 \times 3 \times 5 \times 7,$

and $180 = 2 \times 90 = 2 \times 2 \times 45 = 2 \times 2 \times 3 \times 15 = 2 \times 2 \times 3 \times 3 \times 5.$

The common factors in the three numbers are 3 and 3.

$\therefore$ HCF $= 3 \times 3 = 9.$

Note The HCF of coprime numbers is 1, since they have no other factor in common.

To find the HCF by division

Steps
1. Divide the bigger number by the smaller number.
2. If the division leaves no remainder then the smaller number is the HCF. But if there is a remainder, take this as the new divisor and take the previous divisor as the new dividend.
3. Continue Step 2 until there is no remainder. The last divisor is the required HCF.

EXAMPLE **Find the HCF of 240 and 336.**

Solution Here, 240 < 336. So, we take 240 as the divisor and 336 as the dividend.

```
240) 336 (1                 (Step 1)
   - 240
      96) 240 (2            (Step 2)
        - 192
           48) 96 (2        (Step 2 repeated)
             - 96
                ×
```

$\therefore$ HCF = 48.

To find the HCF of more than two numbers:

Steps
1. Find the HCF of two of the numbers.
2. Find the HCF of the third number and the number obtained in Step 1.
3. Find the HCF of the fourth number and the number obtained in Step 2, and so on.

EXAMPLE **Find the HCF of 256, 940 and 442.**

Solution First, find the HCF of 256 and 940.

```
256) 940 (3
   - 768
     172) 256 (1
        - 172
           84) 172 (2
             - 168
                 4) 84 (21
                  - 8
                    × 4
                    - 4
                      ×
```

$\therefore$ the HCF of 256 and 940 is 4.

Now, find the HCF of 4 and 442.

4) 442 (110
 − 4
 × 4
 − 4
 × 2) 4 (2
 − 4
 ×

∴ the HCF of 4 and 442 is 2.

∴ the HCF of the three numbers is 2.

Note This method is more convenient when the numbers are big.

Solved Examples

EXAMPLE 1 **Find the HCF of 96 and 120.**

Solution *By prime factorisation*

$96 = 2 \times 48 = 2 \times 2 \times 24$
$= 2 \times 2 \times 2 \times 12 = 2 \times 2 \times 2 \times 2 \times 6$
$= 2 \times 2 \times 2 \times 2 \times 2 \times 3.$
$120 = 2 \times 60 = 2 \times 2 \times 30$
$= 2 \times 2 \times 2 \times 15 = 2 \times 2 \times 2 \times 3 \times 5.$

The common factors are 2, 2, 2 and 3.

∴ HCF $= 2 \times 2 \times 2 \times 3 = 24.$

By division

96 < 120. So, 96 is the divisor.

96) 120 (1
 − 96
 24) 96 (4
 − 96
 ×

∴ HCF = 24.

EXAMPLE 2 **Find the HCF of 585, 330 and 420.**

Solution *By prime factorisation*

$585 = 5 \times 117 = 5 \times 3 \times 39 = 5 \times 3 \times 3 \times 13.$
$330 = 2 \times 165 = 2 \times 5 \times 33 = 2 \times 5 \times 3 \times 11.$
$420 = 2 \times 210 = 2 \times 2 \times 105 = 2 \times 2 \times 5 \times 21 = 2 \times 2 \times 5 \times 3 \times 7.$

The common factors of the three numbers are 5 and 3.

∴ HCF $= 5 \times 3 = 15.$

By division

330) 585 (1
 − 330
 255) 330 (1
 − 255
 75) 255 (3
 − 225
 30) 75 (2
 − 60
 15) 30 (2
 − 30
 ×

∴ the HCF of 585 and 330 is 15.

Now,

```
15) 420 (28
   - 30
    120
   - 120
      ×
```

∴ the HCF of 15 and 420 is 15.

∴ the HCF of 585, 330 and 420 is 15.

EXAMPLE 3 **Find the greatest number that will divide 90, 108 and 126 without leaving a remainder.**

Solution The required number is the GCD (or HCF) of the numbers 90, 108 and 126.

$90 = 2 \times 45 = 2 \times 3 \times 15 = 2 \times 3 \times 3 \times 5.$

$108 = 2 \times 54 = 2 \times 2 \times 27 = 2 \times 2 \times 3 \times 9 = 2 \times 2 \times 3 \times 3 \times 3.$

$126 = 2 \times 63 = 2 \times 3 \times 21 = 2 \times 3 \times 3 \times 7.$

The common factors are 2, 3, 3.

∴ the required number is $2 \times 3 \times 3$, that is, 18.

EXAMPLE 4 **Find the greatest number that will divide 137, 182 and 422, leaving the remainder 2 in each case.**

Solution Here, $137 - 2 = 135$, $182 - 2 = 180$, $422 - 2 = 420$.

∴ the required number is the GCD of the numbers 135, 180 and 420.

```
135) 180 (1
   - 135
      45) 135 (3
         -135
            ×
```

∴ the GCD (or HCF) of 135 and 180 is 45.

Now,

```
45) 420 (9
  - 405
     15) 45 (3
        - 45
           ×
```

∴ the GCD (or HCF) of 45 and 420 is 15.

∴ the required number is 15.

Note If the remainders are different then subtract the remainders from the corresponding numbers and then find the HCF of the resulting numbers.

EXAMPLE 5 **Reduce $\frac{357}{525}$ to the lowest terms.**

Solution To reduce the given fraction to the lowest terms, we divide its numerator and denominator by their HCF. Now, we find the HCF of 357 and 525.

```
357) 525 (1
   - 357
     168) 357 (2
        - 336
           21) 168 (8
             - 168
                 ×
```

∴ the HCF of 357 and 525 is 21.

So, $\frac{357}{525} = \frac{357 \div 21}{525 \div 21} = \frac{17}{25}.$

EXAMPLE 6 **Three tankers contain 434 litres, 465 litres and 496 litres of petrol respectively. Find the maximum capacity of a container that can measure the petrol of the three containers exact number of times.**

Solution The maximum capacity of a container which can measure the petrol of the three tankers is same as the HCF of 434 L, 465 L and 496 L.

First, we find the HCF of 434 and 465.

```
434) 465 (1
    - 434
     31) 434 (14
         - 31
          124
        - 124
            ×
```

∴ the HCF of 434 and 465 is 31.

Now, we find the HCF of 31 and 496.

```
31) 496 (16
   - 31
    186
  - 186
      ×
```

∴ the HCF of 31 and 496 is 31.

So, the HCF of 434, 465 and 496 is 31.

Hence, the maximum capacity of the container is 31 L.

Remember These

1. A common factor of two or more numbers divides all the numbers exactly.
2. The HCF, or GCD of two or more numbers is the greatest number which divides all the numbers without leaving a remainder. It can be found by (a) prime factorisation and (b) division.
3. Two numbers are coprimes if their HCF is 1.

EXERCISE 2D

1. Find the HCF of the following by prime factorisation.
   (i) 40 and 96 (ii) 36 and 60 (iii) 81 and 27 (iv) 112 and 160
   (v) 84 and 105 (vi) 216 and 630 (vii) 58 and 174 (viii) 165 and 275
2. Find the GCD of the following by division.
   (i) 390 and 663 (ii) 837 and 1134 (iii) 504 and 5292 (iv) 856 and 936
   (v) 775 and 1800 (vi) 286 and 352 (vii) 7625 and 8175
3. Find the HCF of the following.
   (i) 63, 90 and 36 (ii) 35, 77 and 56 (iii) 40, 48 and 72
   (iv) 44, 121 and 132 (v) 128, 136 and 512 (vi) 345, 726 and 531
   (vii) 432, 1134 and 1347 (viii) 176, 1100 and 4444 (ix) 1326, 3094 and 4420

4. Find the HCF of the following.
   (i) 60, 84, 108 and 132 (ii) 84, 56, 35 and 112
5. (a) Find the greatest number that will divide 58 and 145 exactly.
   (b) Find the largest number that will divide 540, 315 and 360 exactly.
   (c) What is the largest number that has the multiples 168 and 288?
6. (a) What is the greatest number that will divide 33 and 45, leaving 9 as the remainder in each case?
   (b) What is the greatest number that will divide 139, 184 and 424, leaving the remainder 4 in each case?
   (c) Find the greatest number that will divide 37, 56 and 93, leaving the remainder 1, 2 and 3 respectively.
7. Use HCF to prove that the following numbers are coprimes.
   (i) 980 and 1287 (ii) 1365 and 2057
8. Reduce each of the following fractions to the lowest terms.
   (i) $\frac{306}{414}$ (ii) $\frac{559}{989}$ (iii) $\frac{611}{1457}$ (iv) $\frac{350}{518}$
9. Find the greatest length of a tape whose whole length can be used to measure exactly the lengths 56 cm, 154 cm, and 294 cm.
10. There are 342 apples, 627 mangoes and 836 guavas. These are to be arranged in heaps containing the same number of fruits. Find the greatest number of fruits possible in each heap. How many heaps are formed?
11. Find the greatest weight that can be used a number of times to weigh 156 kg, 180 kg and 324 kg of wheat.
12. In a school library, there are 780 books of English, 364 books of Hindi and 624 books of mathematics. The librarian wants to arrange these books across the maximum number of shelves such that each shelf has the same number of books of each subject. Find the number of shelves that can be used. What would be the number of these books in each shelf?

ANSWERS

1. (i) 8 (ii) 12 (iii) 27 (iv) 16 (v) 21 (vi) 18 (vii) 58 (viii) 55
2. (i) 39 (ii) 27 (iii) 252 (iv) 8 (v) 25 (vi) 22 (vii) 25
3. (i) 9 (ii) 7 (iii) 8 (iv) 11 (v) 8 (vi) 3 (vii) 3 (viii) 44 (ix) 442
4. (i) 12 (ii) 7
5. (a) 29 (b) 45 (c) 24
6. (a) 12 (b) 15 (c) 18
8. (i) $\frac{17}{23}$ (ii) $\frac{13}{23}$ (iii) $\frac{13}{31}$ (iv) $\frac{25}{37}$
9. 14 cm **10.** 19, 95 **11.** 12 kg **12.** 52; 34

Lowest Common Multiple (LCM)

Let us consider the two numbers 4 and 6.

The multiples of 4 are $1\times4, 2\times4, 3\times4, 4\times4, 5\times4, 6\times4$, etc.,
that is, 4, 8, (12), 16, 20, (24), etc.

The multiples of 6 are $1\times6, 2\times6, 3\times6, 4\times6, 5\times6, 6\times6$, etc.,
that is, 6, (12), 18, (24), 30, 36, etc.

Clearly, 12 and 24 are multiples which are common to both the numbers. So are 36, 48, and so on. Of all these common multiples, 12 is the smallest. It is called the least common multiple of 4 and 6.

The lowest common multiple (LCM) of two or more numbers is the smallest of the common multiples of those numbers. As a multiple of a number is divisible by the number, the LCM of two or more numbers is the smallest number which can be divided by each of the numbers without leaving a remainder.

The LCM of two or more numbers can be found by (a) prime factorisation and (b) division.

To find the LCM by prime factorisation

Steps
1. Express each number as a product of prime numbers.
2. Take each prime factor as many times as the highest number of times it appears in the prime factorisations of any of the numbers.
3. The product of the prime factors selected in Step 2 is the LCM of the numbers.

EXAMPLE **Find the LCM of 36 and 48.**

Solution On prime factorisation,

$36 = 2 \times 2 \times 3 \times 3$

and $48 = 2 \times 2 \times 2 \times 2 \times 3$ (Step 1).

Here, 2 appears twice in the factorisation of 36 and four times in that of 48. So, select 2 four times. 3 appears twice in the factorisation of 36 and once in that of 48. So, take 3 two times (Step 2).

$\therefore$ LCM $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 144$ (Step 3).

EXAMPLE **Find the LCM of 40, 144 and 180.**

Solution On prime factorisation,

$40 = 2 \times 2 \times 2 \times 5$,

$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$,

and $180 = 2 \times 2 \times 5 \times 3 \times 3$.

The highest number of times 2 appears in the factorisation of any of the numbers is four, 3 appears twice, and 5 appears once.

$\therefore$ LCM $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 720$.

To find the LCM by division

Steps
1. Divide the numbers by a number which is a factor of at least two of the numbers.
2. Write the dividends and carry forward the numbers which are not divisible.
3. Repeat the steps 1 and 2 till no two of the numbers has a common factor.
4. The product of the divisors of all the steps and the remaining numbers is the LCM of the numbers.

EXAMPLE **Find the LCM of 36 and 48.**

Solution

2	36, 48	
2	18, 24	
3	9, 12	
	3, 4	(no common factor)

$\therefore$ LCM $= 2 \times 2 \times 3 \times 3 \times 4 = 144$.

EXAMPLE **Find the LCM of 40, 144 and 180.**

Solution

2	40, 144, 180	
2	20, 72, 90	
2	10, 36, 45	(2 is a common factor of 10, 36)
3	5, 18, 45	(45 carried forward; 3 is a common factor of 18, 45)
3	5, 6, 15	(5 carried forward; 3 is a common factor of 6, 15)
5	5, 2, 5	(5 carried forward; 5 is a common factor of 5, 5)
	1, 2, 1	(no common factor)

$\therefore$ LCM $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 2 = 720$.

Relation between the HCF and LCM of two numbers

The product of two numbers is equal to the product of their HCF and LCM.

HCF $\times$ LCM $=$ (first number) $\times$ (second number)

It follows that:

1. LCM $=$ (product of the two numbers) $\div$ HCF.
2. HCF $=$ (product of the two numbers) $\div$ LCM.
3. HCF $\times$ LCM $\div$ (one number) $=$ other number.
4. LCM of two coprimes $=$ product of numbers, because their HCF is 1.

EXAMPLE **Verify that the product of the HCF and LCM of 48 and 72 is equal to the product of the numbers.**

Solution On prime factorisation,

$48 = 2 \times 2 \times 2 \times 2 \times 3$

and $72 = 2 \times 2 \times 2 \times 3 \times 3$.

The common factors are 2, 2, 2, 3.

$\therefore$ HCF $= 2 \times 2 \times 2 \times 3 = 24$

and LCM $= 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 144$.

Now, HCF $\times$ LCM $= 24 \times 144 = 3456 = 72 \times 48$.

Solved Examples

EXAMPLE 1 **Find the LCM of 75 and 120.**

Solution *By prime factorisation*

$75 = 5 \times 5 \times 3.$

$120 = 2 \times 2 \times 2 \times 3 \times 5.$

$\therefore$ $\text{LCM} = 2 \times 2 \times 2 \times 3 \times 5 \times 5 = 600.$

By division

$$\begin{array}{r|l} 5 & 75,\ 120 \\ \hline 3 & 15,\ 24 \\ \hline \end{array}$$

$\therefore$ $\text{LCM} = 5 \times 3 \times 5 \times 8 = 600.$

EXAMPLE 2 **Find the LCM of 135, 162 and 108.**

Solution *By prime factorisation*

$135 = 5 \times 3 \times 3 \times 3.$

$162 = 2 \times 3 \times 3 \times 3 \times 3.$

$108 = 2 \times 2 \times 3 \times 3 \times 3.$

$\therefore$ $\text{LCM} = 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 = 1620.$

By division

$$\begin{array}{r|l} 2 & 135,\ 162,\ 108 \\ \hline 3 & 135,\ 81,\ 54 \\ \hline 3 & 45,\ 27,\ 18 \\ \hline 3 & 15,\ 9,\ 6 \\ \hline \end{array}$$

$\therefore$ $\text{LCM} = 2 \times 3 \times 3 \times 3 \times 5 \times 3 \times 2.$

$= 1620$

EXAMPLE 3 **Find the smallest number which is exactly divisible by 36, 54, 72 and 27.**

Solution By the definition of LCM, the required number is the LCM of 36, 54, 72 and 27.

$$\begin{array}{r|l} 2 & 36,\ 54,\ 72,\ 27 \\ \hline 2 & 18,\ 27,\ 36,\ 27 \\ \hline 3 & 9,\ 27,\ 18,\ 27 \\ \hline 3 & 3,\ 9,\ 6,\ 9 \\ \hline 3 & 1,\ 3,\ 2,\ 3 \\ \hline & 1,\ 1,\ 2,\ 1 \end{array}$$

$\therefore$ $\text{LCM} = 2 \times 2 \times 3 \times 3 \times 3 \times 2 = 216$, which is the required number.

EXAMPLE 4 **Find the smallest number which when divided by 20, 28 and 36 leaves 4 as the remainder in each case.**

Solution The smallest number which is divisible by 20, 28 and 36 is the LCM of 20, 28 and 36.

$$\begin{array}{r|l} 4 & 20,\ 28,\ 36 \\ \hline & 5,\ 7,\ 9 \end{array}$$

$\therefore$ $\text{LCM} = 4 \times 5 \times 7 \times 9 = 1260.$

When the required number is divided by any of the given numbers, the remainder is 4.

$\therefore$ the required number = (LCM of 20, 28 and 36) + 4 = 1260 + 4 = 1264.

EXAMPLE 5 **Find the LCM of 45 and 75. Use the LCM to find their HCF.**

Solution

$$\begin{array}{r|l} 5 & 45,\ 75 \\ \hline 3 & 9,\ 15 \\ \hline & 3,\ 5 \end{array}$$

$\therefore$ the LCM of 45 and 75 is $5 \times 3 \times 3 \times 5 = 225.$

Now, HCF = (product of the two numbers) ÷ LCM

$= (45 \times 75) \div 225 = 3375 \div 225 = 15.$

EXAMPLE 6 **The LCM and HCF of two numbers are 120 and 12 respectively. If one of the numbers is 24, find the other number.**

Solution (The required number) $\times 24 = \text{LCM} \times \text{HCF} = 120 \times 12$.

$\therefore$ the required number is $(120 \times 12) \div 24 = 1440 \div 24 = 60$.

EXAMPLE 7 **Three bells toll at intervals of 12 minutes, 15 minutes and 18 minutes. If they start tolling together, after what time will they next toll together?**

Solution The bells will next toll together at a time which is the lowest common multiple of 12 min, 15 min and 18 min.

$$\begin{array}{r|l} 2 & 12,\ 15,\ 18 \\ \hline 3 & 6,\ 15,\ 9 \\ \hline & 2,\ 5,\ 3 \end{array}$$

$\therefore$ $\text{LCM} = 2 \times 3 \times 2 \times 5 \times 3 = 180.$

Thus, the bells will toll together after 180 minutes, that is, after 3 hours.

Remember These

1. The LCM of two or more numbers is the smallest number which can be divided by all the numbers. The LCM can be found by (a) prime factorisation or (b) division.
2. The product of two numbers is equal to the product of their HCF and LCM.
3. LCM of two numbers = (product of the numbers) ÷ (HCF of the numbers).
   HCF of two numbers = (product of the numbers) ÷ (LCM of the numbers).
4. Given the HCF and LCM of two numbers and one of the numbers, the other number = (HCF × LCM) ÷ (the given number).

EXERCISE 2E

1. Find the LCM of the following by prime factorisation.
   (i) 25 and 80 (ii) 75 and 120 (iii) 36 and 39
   (iv) 144 and 204 (v) 132 and 84 (vi) 144 and 270
2. Find the LCM of the following by division.
   (i) 32 and 64 (ii) 28 and 70 (iii) 105 and 175
3. Find the LCM of the following.
   (i) 48, 72 and 36 (ii) 42, 56 and 80 (iii) 32, 40 and 84
   (iv) 60, 40 and 90 (v) 24, 36, 54 and 60 (vi) 35, 25, 45 and 75
4. Find the smallest number which is exactly divisible by
   (i) 24, 20 (ii) 32, 40 and 72 (iii) 18, 24, 36 and 42
5. Find the HCF as well as the LCM of each of the following pairs.
   (i) 108 and 180 (ii) 33 and 132 (iii) 90 and 144

6. (a) Find the smallest number which when divided by 45 and 60 leaves 7 as the remainder in each case.

(b) Find the smallest number which when divided by 40, 60 and 100 leaves 24 as the remainder in each case.

(c) Find the smallest number which when divided by 15, 20, 25 and 45 leaves 10 as the remainder in each case.

7. (a) The LCM and HCF of two numbers are 252 and 6 respectively. If one of the numbers is 42, find the other number.

(b) The LCM and HCF of two numbers are 360 and 12 respectively. If the bigger number is 72, find the smaller number.

8. In a morning walk, three persons step off together. Their steps measure 70 cm, 75 cm and 80 cm respectively. What is the minimum distance each should walk so that all can cover the same distance in complete steps?

9. The traffic lights at four different road crossings change after every 24 seconds, 48 seconds, 72 seconds and 108 seconds respectively. If they change simultaneously at 8 a.m., at what time will they change simultaneously again?

10. Three measuring rods are 50 cm, 60 cm, and 70 cm in length. What is the least length of a rope that can be measured by the full length of each of these three rods?

ANSWERS

1. (i) 400 (ii) 600 (iii) 468 (iv) 2448 (v) 924 (vi) 2160

2. (i) 64 (ii) 140 (iii) 525

3. (i) 144 (ii) 1680 (iii) 3360 (iv) 360 (v) 3240 (vi) 1575

4. (i) 120 (ii) 1440 (iii) 504

5. (i) HCF = 36, LCM = 540 (ii) HCF = 33, LCM = 132 (iii) HCF = 18, LCM = 720

6. (a) 187 (b) 624 (c) 910 **7.** (a) 36 (b) 60 **8.** 84 m

9. 7 minutes 12 seconds past 8 a.m. **10.** 21 m

❖

3

Whole Numbers

Natural numbers and whole numbers

You are familiar with the numbers used to count things, i.e. 1, 2, 3, 4, etc. These counting numbers are called natural numbers. The smallest natural number is 1. There is no greatest natural number, since the list of natural numbers is infinite or endless.

The number zero, used to count 'nothing' is not a natural number. However, it is a whole number, as are all the natural numbers. Thus, the numbers 1, 2, 3, 4, ... are natural number, while the numbers 0, 1, 2, 3, 4, ... are whole numbers. The smallest whole number is 0.

> All the natural numbers are also whole numbers. Zero (0) is a whole number but not a natural number.

Successor and predecessor of a whole number

Given a whole number, the whole number greater than the given number by 1 is its successor. The whole number less than the given number by 1 is its predecessor.

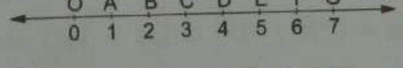
Thus,	(a whole number) + 1 is the successor of the whole number
and	(a whole number) – 1 is the predecessor of the whole number.

Clearly, every whole number has a successor and every whole number other than 0 has a predecessor. 0 has no predecessor.

Representation of whole numbers on the number line

The whole numbers can be represented on a line in the following way.

1. Draw a straight line l. Mark a point O on it and label the point as 0 (zero).



2. Select a unit of length. Let it be 1 cm. Mark a point A on the line l on the right of O such that OA = 1 cm. Label the point A as 1 (one).
3. Mark another point B on the right of A such that AB = 1 cm. Label the point as 2 (two).
4. Mark points C, D, E, ... consecutively on the right of B such that each of BC, CD, DE, ... has the length 1 cm. Label the points C, D, E, ... as 3, 4, 5, ...

The line l on which the whole numbers are thus represented is called the number line.

Fundamental Operations on Whole Numbers

You are familiar with the operations of addition, subtraction, multiplication and division on whole numbers. Here, we study some properties of these operations.

Properties of addition

Closure property

The sum of any two whole numbers is always a whole number.

Examples (i) $7 + 27 = 34$, a whole number.

(ii) $18 + 96 = 114$, a whole number.

(iii) $316 + 580 = 896$, a whole number.

(iv) $0 + 6842 = 6842$, a whole number.

Thus, if a and b are whole numbers then $a + b$ is a whole number.

Commutative property

The sum of two whole numbers remains the same irrespective of the order in which we add them.

Symbolically, $a + b = b + a$.

Examples (i) $167 + 352 = 519$, $352 + 167 = 519 \Rightarrow 167 + 352 = 352 + 167$.

(ii) $2356 + 6789 = 9145$, $6789 + 2356 = 9145$

$\Rightarrow 2356 + 6789 = 6789 + 2356$.

Associative property

If we have to find the sum of three whole numbers, we can add *any* two of the numbers and then add the third number to their sum.

Symbolically, $(a + b) + c = a + (b + c) = a + b + c$.

Examples (i) $(26 + 45) + 67 = 71 + 67 = 138$.

$26 + (45 + 67) = 26 + 112 = 138$.

$\therefore \quad (26 + 45) + 67 = 26 + (45 + 67)$.

(ii) $(678 + 223) + 957 = 901 + 957 = 1858$.

$678 + (223 + 957) = 678 + 1180 = 1858$.

$\therefore \quad (678 + 223) + 957 = 678 + (223 + 957)$.

Calculations often become easier when we use these properties.

Examples (i) $643 + 598 + 457 = 643 + (598 + 457)$

$= 643 + (457 + 598)$

(by the commutative property)

$= (643 + 457) + 598$

(by the associative property)

$= 1100 + 598 = 1698.$

(ii) $336 + 623 + 584 + 577 = (336 + 584) + (623 + 577)$

$= 920 + 1200 = 2120.$

Zero (0) is called the additive identity. For any whole number a, we can write $a + 0 = 0 + a = a$.

Properties of subtraction

Closure property

Subtraction of a whole number from an equal whole number or from a larger whole number always gives a whole number.

Examples (i) $12 - 7 = 5$, a whole number.

(ii) $84 - 84 = 0$, a whole number.

(iii) $3571 - 3507 = 64$, a whole number.

(iv) $123 - 217$ is not defined in whole numbers.

Thus, if a and b are two whole numbers and either $a > b$ or $a = b$ then $a - b$ is a whole number.

Now, $7 - 5 = 2$ but $5 - 7$ is not defined in whole numbers.

So, if a and b are any two whole numbers then $a - b \neq b - a$.

So, commutative property does not hold for subtraction.

Also, $9 - (4 - 2) = 9 - 2 = 7$

but $(9 - 4) - 2 = 5 - 2 = 3$.

So, $9 - (4 - 2) \neq (9 - 4) - 2$.

So, if a, b, c are any three whole numbers then in general,

$(a - b) - c \neq a - (b - c)$.

Thus, in general, the associative property does not hold for subtraction.

Also, $12 - 0 = 12$ but $0 - 12$ is not defined in whole numbers.

Properties of multiplication

Closure property

The product of any two whole numbers is always a whole number.

> If a and b are whole numbers, $a \times b$ is also a whole number.

Examples (i) $6 \times 17 = 102$, a whole number.

(ii) $19 \times 105 = 1995$, a whole number.

Commutative property

The product of two whole numbers remains the same irrespective of the order in which we multiply them.

Symbolically, $a \times b = b \times a$.

Examples (i) $24 \times 78 = 1872$, $78 \times 24 = 1872$.
$\therefore$ $24 \times 78 = 78 \times 24$.

(ii) $125 \times 246 = 30750$, $246 \times 125 = 30750$.
$\therefore$ $125 \times 246 = 246 \times 125$.

Associative property

To find the product of three whole numbers, we first multiply *any* two of them and then multiply the product by the third number. It does not matter which two numbers we multiply first.

Symbolically, $a \times b \times c = (a \times b) \times c = a \times (b \times c)$.

Examples (i) $6 \times 7 \times 8 = (6 \times 7) \times 8 = 42 \times 8 = 336$.
Changing the arrangement, $6 \times 7 \times 8 = 6 \times (7 \times 8) = 6 \times 56 = 336$.
Thus, $(6 \times 7) \times 8 = 6 \times (7 \times 8) = 6 \times 7 \times 8$.

(ii) $(12 \times 15) \times 17 = 180 \times 17 = 3060$, $12 \times (15 \times 17) = 12 \times 255 = 3060$.
$\therefore$ $(12 \times 15) \times 17 = 12 \times (15 \times 17)$.

Sometimes these properties of multiplication make calculations easier.

Examples (i) $15 \times 137 \times 40 = 15 \times (137 \times 40)$
$= 15 \times (40 \times 137)$ (by the commutative property)
$= (15 \times 40) \times 137$ (by the associative property)
$= 600 \times 137 = 82200$.

(ii) $12359 \times 36 \times 27 \times 25 = (12359 \times 27) \times (36 \times 25)$
$= 333693 \times 900 = 300323700$.

Distributive property

If a, b and c are three whole numbers then

$a \times (b + c) = a \times b + a \times c$ and $a \times (b - c) = a \times b - a \times c$.

Examples (i) $67 \times 32 = 67 \times (30 + 2) = 67 \times 30 + 67 \times 2 = 2010 + 134 = 2144$.

(ii) $56 \times 98 = 56 \times (100 - 2) = 56 \times 100 - 56 \times 2 = 5600 - 112 = 5488$.

The number 1 is called the multiplicative identity. For any whole number a, we have $a \times 1 = 1 \times a = a$.

Operation of division

You know how to divide a whole number by a smaller nonzero whole number. For example, if 7 is divided by 2, the quotient is 3 and the remainder is 1. This means that $7 = 2 \times 3 + 1$. We can generalise this result for any two nonzero whole numbers.

Dividend = divisor × quotient + remainder.

We can put this in another way. If a is a whole number and b is a smaller nonzero whole number then there exist two other whole numbers q and r such that

$$a = bq + r, \text{ where } r = 0 \text{ or } r < b.$$

This relation is called the division algorithm or the rule of division.

Properties of division

9 and 2 are whole numbers but $9 \div 2$ is not a whole number. So, if a and b be any two nonzero whole numbers then $a \div b$ is not always a whole number.

Also, if a be any whole number then $a \div 0$ is not defined. If a be any nonzero whole number then $0 \div a = 0$.

Examples (i) $0 \div 2 = 0$.

(ii) $0 \div 13 = 0$.

Solved Examples

EXAMPLE 1 **Find each of the following products using the distributive law.**

(i) 4123×9 (ii) 6234×99 (iii) 786×999

Solution (i) $4123 \times 9 = 4123 \times (10 - 1) = 4123 \times 10 - 4123 \times 1$
$= 41230 - 4123 = 37107.$

(ii) $6234 \times 99 = 6234 \times (100 - 1)$
$= 6234 \times 100 - 6234 \times 1$
$= 623400 - 6234 = 617166.$

(iii) $786 \times 999 = 786 \times (1000 - 1)$
$= 786 \times 1000 - 786 \times 1$
$= 786000 - 786 = 785214.$

EXAMPLE 2 **Find the value of $847 \times 67 + 847 \times 33$.**

Solution $847 \times 67 + 847 \times 33 = 847 \times (67 + 33)$ (by the distributive property)
$= 847 \times 100 = 84700.$

EXAMPLE 3 **Find the value of $2301 \times 745 + 2301 \times 167 + 2301 \times 88$.**

Solution $2301 \times 745 + 2301 \times 167 + 2301 \times 88$
$= 2301 \times (745 + 167 + 88)$
$= 2301 \times 1000 = 2301000.$

EXAMPLE 4 **Simplify: $7813 \times 147 - 47 \times 7813$.**

Solution $7813 \times 147 - 47 \times 7813$

$= 7813 \times 147 - 7813 \times 47$ (by the commutative property)

$= 7813 \times (147 - 47)$ (by the distributive property)

$= 7813 \times 100 = 781300$.

EXAMPLE 5 **Divide 2907 by 54 and verify the division algorithm.**

Solution

```
54) 2907 (53
  - 270
     207
   - 162
      45
```

The division algorithm is $a = bq + r$, where $r = 0$ or $r < b$.

Here, $a = 2907$, $b = 54$, $q = 53$, $r = 45$.

Now, $bq + r = 54 \times 53 + 45 = 2862 + 45 = 2907 = a$.

$\therefore$ the division algorithm is satisfied.

EXAMPLE 6 **Find the number which when divided by 35 gives the quotient 20 and the remainder 18.**

Solution The required number is divisor × quotient + remainder $= 35 \times 20 + 18 = 718$.

EXAMPLE 7 **Find the largest five-digit number which is exactly divisible by 47.**

Solution The largest five-digit number is 99999.

```
47) 99999 (2127
   - 94
      59
    - 47
     129
    - 94
     359
   - 329
      30
```

99999 divided by 47 gives the remainder 30.

Hence, the required number

$= 99999 -$ remainder

$= 99999 - 30 = 99969$.

EXAMPLE 8 **Find the smallest six-digit number which is exactly divisible by 175.**

Solution The smallest six-digit number is 100000.

```
175) 100000 (571
    - 875
     1250
   - 1225
      250
    - 175
       75
```

In order to be divisible by 175, the required number must be greater than 100000 by $175 - 75 = 100$.

Thus, the required number $= 100000 + 100 = 100100$.

EXERCISE 3A

1. Fill in the blanks.

(i) ...... $\times (65 - 15) = 35 \times 65 - 35 \times 15$

(ii) $357 +$ ...... $= 758 + 357$

(iii) $6142 + (1236 +$ ......$) = (6142 +$ ......$) + 3487$

(iv) $68 \times ($...... $+$ ......$) = 68 \times 36 + 68 \times 64$

2. Find the sum of the following numbers and verify the commutative law of addition.
   (i) 256, 479 (ii) 6379, 12676 (iii) 52974, 2765

3. Find the sum of the following numbers, using the most convenient grouping.
   (i) 3719, 286, 5281 (ii) 1952, 357, 2448, 743
   (iii) 1841, 4477, 3459, 2673, 1223

4. Find the following products and verify the commutative property of multiplication.
   (i) 126×63 (ii) 437×742

5. Find the products using the most convenient grouping.
   (i) $8 \times 427 \times 25$ (ii) $2754 \times 5 \times 21 \times 20$ (iii) $327 \times 125 \times 16 \times 12$

6. Find the products using the distributive property.
   (i) 231×11 (ii) 267×9 (iii) 348×1001
   (iv) 7084×99 (v) 9830×999 (vi) 34627×9999

7. Find the following using relevant property.
   (i) $784 \times 117 - 784 \times 17$ (ii) $3056 \times 92 + 3056 \times 8$
   (iii) $93078 \times 1786 - 93078 \times 786$ (iv) $2416 \times 833 + 167 \times 2416$
   (v) $322 \times 678 + 205 \times 322 + 322 \times 117$ (vi) $983 \times 7923 + 2077 \times 983$

8. Divide and verify the answer by division algorithm.
   (i) $4152 \div 257$ (ii) $21486 \div 329$ (iii) $17658 \div 255$

9. Find the number which when divided by 49 gives the quotient 45 and remainder 47.

10. Find the largest six-digit number which is exactly divisible by 127.

11. Find the smallest five-digit number which is exactly divisible by 273,

12. Supply the missing digits.

(i)
```
  8 * 7 *
+ 3 2 * 4
---------
* * 2 3 0
```

(ii)
```
  8 9 4 5
- * * 3 2
---------
  6 3 * *
```

(iii)
```
  235
×   *
-----
* *10
```

ANSWERS

1. (i) 35 (ii) 758 (iii) 3487, 1236 (iv) 36, 64 2. (i) 735 (ii) 19055 (iii) 55739

3. (i) 9286 (ii) 5500 (iii) 13673 4. (i) 7938 (ii) 324254 5. (i) 85400 (ii) 5783400 (iii) 7848000

6. (i) 2541 (ii) 2403 (iii) 348348 (iv) 701316 (v) 9820170 (vi) 346235373

7. (i) 78400 (ii) 305600 (iii) 93078000 (iv) 2416000 (v) 322000 (vi) 9830000

8. (i) quotient = 16, remainder = 40 (ii) quotient = 65, remainder = 101 (iii) quotient = 69, remainder = 63

9. 2252 10. 999998 11. 10101

12. (i)
```
  8976
+ 3254
------
 12230
```

(ii)
```
  8945
- 2632
------
  6313
```

(iii)
```
  235
×   6
-----
 1410
```

Pattern Observations

Observation of patterns can help us in simplifying the operations on numbers.

Addition and subtraction with 9, 99, 999, 9999, ...

Examples (i) $128 + 9 = 128 + 10 - 1 = 138 - 1 = 137$

$128 + 99 = 128 + 100 - 1 = 228 - 1 = 227$

$1754 + 999 = 1754 + 1000 - 1 = 2754 - 1 = 2753$.

$23456 + 9999 = 23456 + 10000 - 1 = 33456 - 1 = 33455$.

And so on.

(ii) $128 - 9 = 128 - 10 + 1 = 118 + 1 = 119$.

$128 - 99 = 128 - 100 + 1 = 28 + 1 = 29$.

$1754 - 999 = 1754 - 1000 + 1 = 754 + 1 = 755$.

$23456 - 9999 = 23456 - 10000 + 1 = 13456 + 1 = 13457$.

And so on.

Multiplication with 5, 25 (or 5 × 5), 125 (or 5 × 5 × 5), 625 (or 5 × 5 × 5 × 5), ...

$32 \times 5 = 32 \times \frac{10}{2} = 16 \times 10 = 160$.

$32 \times 25 = 32 \times \frac{100}{4} = 8 \times 100 = 800$.

$32 \times 125 = 32 \times \frac{1000}{8} = 4 \times 1000 = 4000$.

$32 \times 625 = 32 \times \frac{10000}{16} = 2 \times 10000 = 20000$.

And so on.

Solved Examples

EXAMPLE 1 **Observe the following pattern.**

$$32 \times 5 = 32 \times \frac{10}{2} = 16 \times 10 = 160 \times 1$$

$$32 \times 15 = 32 \times \frac{30}{2} = 16 \times 30 = 160 \times 3$$

$$32 \times 25 = 32 \times \frac{50}{2} = 16 \times 50 = 160 \times 5$$

Now, find (i) 32 × 35 and (ii) 32 × 55.

Solution (i) $32 \times 35 = 32 \times \frac{70}{2} = 16 \times 70 = 160 \times 7$.

(ii) $32 \times 55 = 32 \times \frac{110}{2} = 16 \times 110 = 160 \times 11$.

EXAMPLE 2 **Study the following pattern.**

$$1 = \frac{1\times 2}{2}$$

$$1+2 = \frac{2\times 3}{2}$$

$$1+2+3 = \frac{3\times 4}{2}$$

$$1+2+3+4 = \frac{4\times 5}{2}$$

Now, find (i) $1+2+3+4+5$ and (ii) $1+2+3+4+5+6+7+8+9+10$.

Solution Here, $\text{sum} = \frac{(\text{last term})\times(\text{last term}+1)}{2}$

or $\text{sum} = \frac{(\text{no. of terms})\times(\text{no. of terms}+1)}{2}$.

(i) $1+2+3+4+5 = \frac{5\times(5+1)}{2} = \frac{5\times 6}{2} = 15.$

(ii) $1+2+3+4+5+6+7+8+9+10 = \frac{10\times(10+1)}{2} = \frac{10\times 11}{2} = 55.$

EXERCISE 3B

1. Find using patterns:
   (i) $7854+99$ (ii) $91234+999$ (iii) $30641+9999$ (iv) $104783+99999$
2. Find using patterns:
   (i) $731-9$ (ii) $4752-99$ (iii) $198754-999$ (iv) $71236-9999$
3. Find using patterns:
   (i) 144×5 (ii) 264×25 (iii) 488×125 (iv) 96×625
4. Observe the following pattern:

$$1\times 8+1=9$$
$$12\times 8+2=98$$
$$123\times 8+3=987$$
$$1234\times 8+4=9876$$

   Now, find the following.
   (i) $12345\times 8+5$ (ii) $1234567\times 8+7$ (iii) $123456789\times 8+9$
5. Observe the following pattern:

$$1=1$$
$$11+1=12$$
$$111+11+1=123$$
$$1111+111+11+1=1234$$

   Now, find the following.
   (i) $11111+1111+111+11+1$ (ii) $111111+11111+1111+111+11+1$

6. Observe the following pattern:

$$2 = 1 \times 2$$
$$2 + 4 = 2 \times 3$$
$$2 + 4 + 6 = 3 \times 4$$
$$2 + 4 + 6 + 8 = 4 \times 5$$

Now, find the following:

(i) $2 + 4 + 6 + 8 + 10$ (ii) $2 + 4 + 6 + 8 + 10 + 12 + 14$

ANSWERS

1. (i) 7953 (ii) 92233 (iii) 40640 (iv) 204782 **2.** (i) 722 (ii) 4653 (iii) 197755 (iv) 61237
3. (i) 720 (ii) 6600 (iii) 61000 (iv) 60000 **4.** (i) 98765 (ii) 9876543 (iii) 987654321
5. (i) 12345 (ii) 123456 **6.** (i) $5 \times 6 = 30$ (ii) $7 \times 8 = 56$