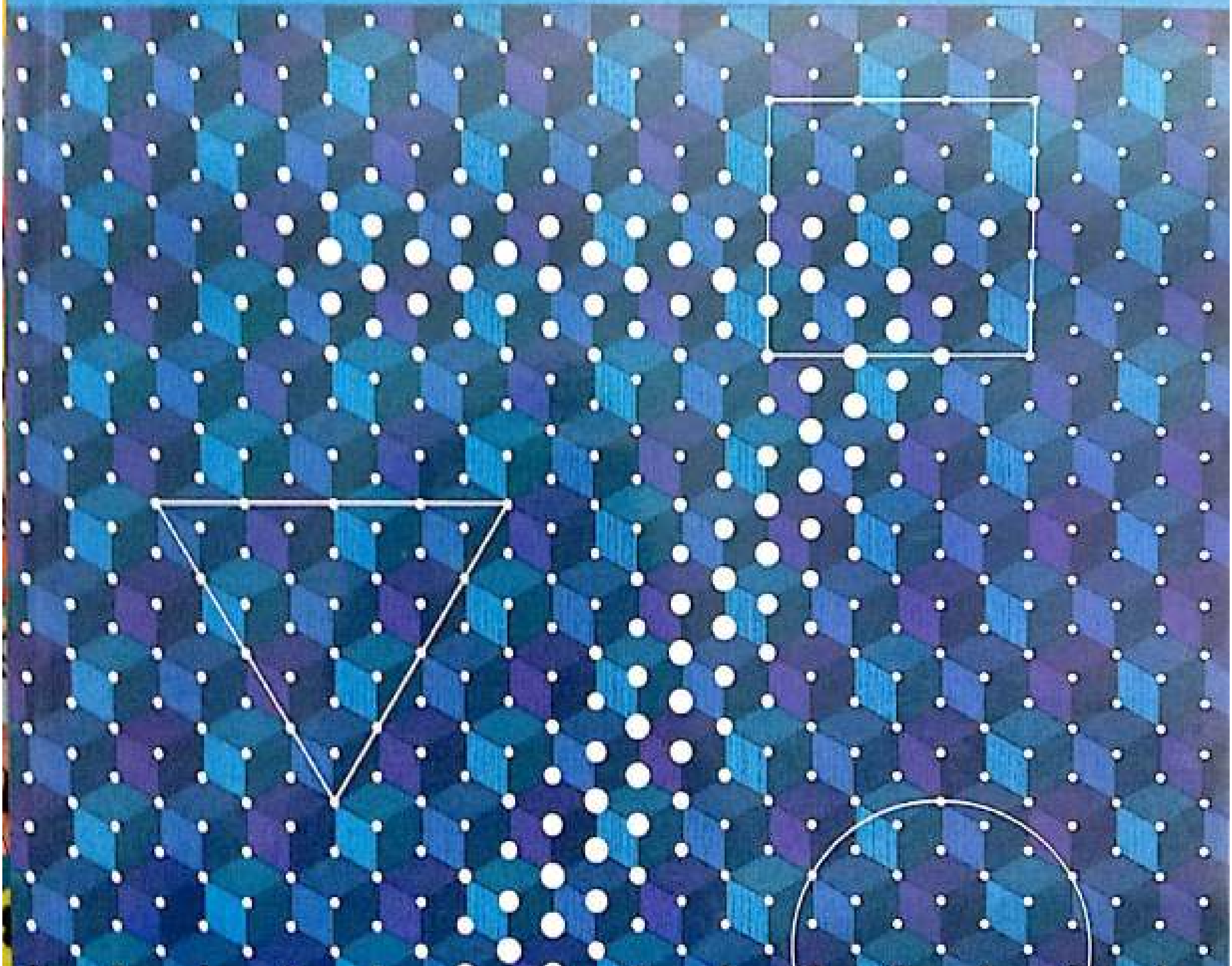
ESSENTIAL ICSE

MATHEMATICS

FOR CLASS 7



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CONTENTS

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1

Concepts of Sets

You are already familiar with the concept of sets. In this chapter and the next, we will review what you have learnt in your previous class and go a little farther from there.

Definition of a set

A set is a collection of well-defined and distinct objects. Every object of the collection forming a set is called a member, or an element, of the set. When an object is a member of a set, we say that the object belongs to the set.

Clearly, any collection of objects is not a set. Take the collections of objects given below.

(i) The first four natural numbers

(ii) Four consecutive natural numbers

(iii) The collection of the letters of the word SCHOOL

The members of the collection (i) are 1, 2, 3 and 4. These objects are well-defined. We are in a position to decide whether a particular object belongs to the collection or not. For example, 3 is a member of the collection, while 5 is not. Also, the members of the collection are distinct or different from each other. Thus, this collection is a set.

But the collection (ii) is not a set. We are unable to decide whether an object belongs to the collection or not. For example, 4 may belong to the collection or it may not. The same is true for any of the other natural numbers.

The collection (iii) is not a set either because the letter O occurs in it twice, and hence the objects of the collection are not distinct. However, if we take only one O in place of two Os, we get the collection of the letters S, C, H, O, L, which is a set.

Notation

Usually, sets are denoted by capital letters, such as A, B, C, X and Y. The members, or elements, are denoted by small letters, such as a, b, c, x and y. If a is a member of the set A, we write it as $a \in A$, which is read as "a belongs to A". If a is not a member of the set A, we write it as $a \notin A$, which is read as "a does not belong to A".

Example Let X be the set of the first five even natural numbers. Then $2 \in X$, $3 \notin X$, $10 \in X$, $12 \notin X$.

Notations for some special sets

(i) The set of natural numbers is denoted by N. As 8 is a natural number, $8 \in N$. But -5 is not a natural number. So, $-5 \notin N$.

(ii) The set of whole numbers is denoted by W. As 0 is a whole number, $0 \in W$. But $\frac{1}{2}$ is not a whole number. So, $\frac{1}{2} \notin W$.

(iii) The set of integers is denoted by I or Z. As -3 is an integer, $-3 \in Z$. But $\frac{3}{5}$ is not an integer. So, $\frac{3}{5} \notin Z$.

Representation of a set

A set can be represented in either of the following two ways.

(i) Roster method or tabular form

(ii) Rule method or set-builder form

Roster method or tabular form

In this method, the members of a set are listed inside the braces, i.e. { }, and separated from each other by commas. If an object appears more than once in a collection, we write it only once.

Examples (i) The set A of the first four natural numbers is written as $A = \{1, 2, 3, 4\}$.

(ii) The set X of the letters of the word KOLKATA is written as $X = \{K, O, L, A, T\}$.

Note The members of a set can be listed in any order. Thus, the set {1, 2, 3, 4} can be written as {2, 3, 4, 1}.

Rule method or set-builder form

If the members of a set have a common property then they can be determined by describing the property. For example, the members of the set $A = \{1, 2, 3, 4, 5\}$ have a common property, namely, all are natural numbers less than 6. No other natural number has this property. So, we can write the set A as follows.

$$A = \{x \mid x \text{ is a natural number less than } 6\},$$

which is read as "A is the set of members x such that x is a natural number less than 6."

This can also be written more precisely as

$$A = \{x \mid x \in N, x < 6\}.$$

Sometimes the set A is also written as

$$A = \{\text{all natural numbers less than } 6\}.$$

In this case, the description of the common property of the members is given inside the braces, i.e. { }. This is a crude form of the rule method.

A set given in the set-builder form can be expressed in the tabular form by listing the objects satisfying the rule.

Examples (i) Let $X = \{x \mid x$ is a letter of the word PATNA$\}$.

Here, x is any one of the letters P, A, T and N; so $X = \{P, A, T, N\}$.

(ii) Let $A = \{x \mid x$ is a whole number greater than 4 but less than 10$\}$.

x is any one of the numbers 5, 6, 7, 8 and 9; so $A = \{5, 6, 7, 8, 9\}$.

A set given in the tabular form can be expressed in the set-builder form if the members obey some common property. But, the representation may not be unique.

Examples (i) Let $B = \{0, 2, 4\}$.

Here, each of the members is an even whole number that is less than 5. No other whole number has this property.

So, $B = \{x \mid x$ is an even whole number less than 5$\}$

or $B = \{x \mid x$ is one of the first three even whole numbers$\}$.

(ii) Let $Y = \{1, e, -3, p\}$.

The members of Y do not have a common property. So, it is difficult to write the set by the rule method.

Types of sets

Finite set

A set is called a finite set if the members of the set can be counted.

Examples (i) $A = \{1, 2, 3, 4\}$, which has four members

(ii) $B = \{x \mid x \in N, x < 10\}$, which has nine members

(iii) $C = \{x \mid x$ is a letter of the English alphabet$\}$, which has 26 members

Infinite set

A set is called an infinite set if it has countless members.

Examples (i) The set W of whole numbers

(ii) The set N of natural numbers

(iii) $O = \{x \mid x$ is an odd natural number$\}$

It is not easy to write infinite sets in the tabular form because it is not possible to make a list of an infinite number of members. However, we do write special sets like N, W and Z in the tabular form as given below.

$$N = \{1, 2, 3, 4, \ldots\},$$
$$W = \{0, 1, 2, 3, \ldots\},$$
$$Z = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}.$$

Similarly, the set of odd positive integers can be written as $\{1, 3, 5, 7, \ldots\}$.

Empty set

The set which has no members is called the empty set or null set. The empty set is denoted by $\varnothing$ or $\{\ \}$.

Examples (i) The set $\{x \mid 2x + 1 = 0$, where $x \in W\}$ represents the empty set because

$$2x + 1 = 0 \ \Rightarrow\ 2x = -1 \ \Rightarrow\ x = -\frac{1}{2} \notin W.$$

(ii) The set $\{x \mid 3x + 13 = 10$, where $x \in N\}$, too, represents the empty set because

$$3x + 13 = 10 \Rightarrow 3x = -3 \Rightarrow x = -1 \notin N.$$

Universal set

The set of all objects under consideration is the universal set for that discussion. For example, if A, B, C, etc., are the sets in our discussion then a set which has all the members of A, B, C, etc., can act as the universal set. Clearly, the universal set varies from problem to problem. It is denoted by U or ξ.

Example If the sets involved in a discussion are sets of some natural numbers then the set N of all natural numbers can be taken as the universal set. The set W of all whole numbers can also be taken as the universal set because all natural numbers are whole numbers.

Cardinal number of a set

The cardinal number of a finite set A is the number of distinct members of the set and it is denoted by $n(A)$. The cardinal number of the empty set, $\emptyset$, is 0 because $\emptyset$ has zero members. So, $n(\emptyset) = 0$. And the cardinal number of an infinite set cannot be found because such a set has countless members.

It is easier to find $n(A)$ if the set A is written in the tabular form, as shown.

Examples (i) If $A = \{-3, -2, -1, 0, 1, 2, 3\}$ then $n(A) = 7$.

(ii) If $A = \{x \mid x$ is a letter of the word HYDERABAD$\}$ then $n(A) = 7$ because writing in the tabular form, $A = \{$H, Y, D, E, R, A, B$\}$.

If A be a set for which $n(A) = 1$, we call the set A a singleton.

If $n(A) = 2$, we call the set A a pair set.

Equivalent sets

Two finite sets with an equal number of members are called equivalent sets. If the sets A and B are equivalent, we write $A \leftrightarrow B$ and read this as "A is equivalent to B".

$$A \leftrightarrow B \text{ if } n(A) = n(B).$$

Examples (i) Let $A = \{0, 2, 4\}$ and $B = \{x \mid x$ is a letter of the word DOOR$\}$.
Then, $n(A) = 3$ and $n(B) = 3$ because $B = \{$D, O, R$\}$. So, $A \leftrightarrow B$.

(ii) Let $X = \{2, 4, 6, 8, 12\}$ and $Y = \{y \mid y$ is a vowel$\}$.
Then, $n(X) = 5$ and $n(Y) = 5$. So, $X \leftrightarrow Y$.

Equal sets

Two sets A and B are said to be equal if every member of A is a member of B, and every member of B is a member of A. This is denoted as $A = B$.

Example Let $A = \{2, 4, 6\}$, $B = \{x \mid x$ is an even natural number less than 7$\}$ and $C = \{x \mid x = 2n, n \in W, n \leq 3\}$.
Writing in the tabular form, $B = \{2, 4, 6\}$ and $C = \{0, 2, 4, 6\}$.

Thus, every member of A is a member of B.

Also, every member of B is a member of A. So, $A = B$.

Again, every member of A is a member of C. But $0 \in C$ and $0 \notin A$. So, every member of C is not a member of A. Hence, $A \neq C$.

Solved Examples

EXAMPLE 1 **Which of the following collections are sets? Give reasons.**

(i) All even whole numbers less than 9

(ii) All the beautiful girls of your school

Solution (i) This is a set because it is possible to decide whether or not a number is a member of the collection.

(ii) This is not a set, as there is no definite way to decide whether a particular girl is beautiful or not.

EXAMPLE 2 **Let X be the set of the natural numbers which are multiples of 3 and are less than 12. Fill in the blanks using $\in$ or $\notin$.**

(i) 6 ...... X (ii) 10 ...... X (iii) 12 ...... X.

Solution (i) $\in$ ($\because$ 6 is a multiple of 3 and it is less than 12.)

(ii) $\notin$ ($\because$ 10 is not a multiple of 3.)

(iii) $\notin$ ($\because$ 12 is not less than 12 though it is a multiple of 3.)

EXAMPLE 3 **Write the following sets in the tabular as well as the set-builder form.**

(i) The set A of odd natural numbers lying between 4 and 10

(ii) The set X of vowels of the word COLLECTION.

Solution (i) In the tabular form: $A = \{5, 7, 9\}$

In the set-builder form: $A = \{x \mid x = 2n - 1, n \in N, 3 \leq n \leq 5\}$.

(ii) In the tabular form: $X = \{O, E, I\}$

In the set-builder form: $X = \{x \mid x$ is a vowel in the word COLLECTION$\}$.

EXAMPLE 4 **(i) Let $A = \{x \mid x = 2n, n \in N, 3 \leq n \leq 7\}$. Write A by the roster method and find $n(A)$.**

(ii) Write the set $B = \{1, 3, 5, 7, 9, 11\}$ by the rule method.

Solution (i) $\because$ $3 \leq n \leq 7$ and $n \in N$, we get $n = 3, 4, 5, 6, 7$.

$\therefore$ $x = 2 \times 3, 2 \times 4, 2 \times 5, 2 \times 6, 2 \times 7 = 6, 8, 10, 12, 14$.

$\therefore$ $A = \{6, 8, 10, 12, 14\}$. As there are exactly five members in A, $n(A) = 5$.

(ii) Each member of B is an odd natural number and no number is greater than 11. So, $x \in B$ if $x \in N$, x is odd and $x \leq 11$.

$\therefore$ $B = \{x \mid x = 2n - 1, n \in N, 1 \leq n \leq 6\}$.

EXAMPLE 5 **Identify the finite and infinite sets among the following.**

(i) $A = \{1, 3, 9, 27\}$ **(ii) $B = \{1, 4, 7, 10, \ldots\}$**

(iii) $C = \{x \mid x = 2n, n \in N, 1 < n < 5\}$ **(iv) $D = \{x \mid x = 2n - 1, n \in W\}$**

Solution (i) A has four members; so it is a finite set.

(ii) The set B is infinite because by adding 3 to a member, we get the next member, and the process is unending.

(iii) In the tabular form, $C = \{4, 6, 8\}$. So, C is a finite set.

(iv) As W is an infinite set, D has countless members. So, D is an infinite set.

EXAMPLE 6 **Identify the empty set, the singleton and the pair set.**

(i) $A = \{x \mid x + 3 = 2, x \in Z\}$ **(ii)** $B = \{x \mid x^2 + 1 = 0, x \in N\}$ **(iii)** $C = \{x \mid x^2 = 4, x \in Z\}$

Solution (i) $x + 3 = 2$ implies that $x = 2 - 3 = -1 \in Z$. So, $A = \{-1\}$ and $n(A) = 1$.

So, A is a singleton.

(ii) $x^2 + 1 \neq 0$ for any natural number x. So, the set B has no members.

$\therefore$ $n(B) = 0$. Therefore, B is the empty set.

(iii) $x^2 = 4 \Rightarrow x = 2, -2$, and $2 \in Z, -2 \in Z$. So, $C = \{-2, 2\}$ and $n(C) = 2$.

So, C is a pair set.

EXAMPLE 7 **Let $X = \{x \mid x$ is a letter of the word LATA$\}$, $Y = \{x \mid x$ is a letter of the word LATE$\}$ and $Z = \{T, A, L, E\}$. Prove that**

(i) $X \neq Y$ **(ii)** $Y \leftrightarrow Z$ **(iii)** $Y = Z$.

Solution Writing in the tabular form,

$$X = \{L, A, T\}, Y = \{L, A, T, E\} \quad \text{and} \quad Z = \{T, A, L, E\}.$$

(i) Every member of X is a member of Y. On the other hand, since $E \in Y$ but $E \notin X$, every member of Y is not a member of X.

$\therefore$ $X \neq Y$.

(ii) $n(Y) = 4$ and $n(Z) = 4$. So, $n(Y) = n(Z) = 4$. So, $Y \leftrightarrow Z$.

(iii) Every member of Y is a member of Z, and every member of Z is a member of Y.

$\therefore$ $Y = Z$.

EXAMPLE 8 **Write the set $A = \{x \mid 2x < 9\}$ in the tabular form, when the universal set is (i) N, (ii) W and (iii) Z.**

Solution (i) As every member of A must belong to the universal set N, x must be a natural number satisfying $2x < 9$. So, $x = 1, 2, 3, 4$ only.

$\therefore$ $A = \{1, 2, 3, 4\}$.

(ii) As every member of A must belong to the universal set W, x must be a whole number satisfying $2x < 9$. So, $x = 0, 1, 2, 3, 4$ only.

$\therefore$ $A = \{0, 1, 2, 3, 4\}$.

(iii) As every member of A must belong to the universal set Z, x must be an integer satisfying $2x < 9$. So, x = any negative integer, 0, 1, 2, 3, 4.

$\therefore$ $A = \{\ldots, -2, -1, 0, 1, 2, 3, 4\}$.

Remember These

1. A collection of distinct objects is a set if one can decide whether a particular object is a member of the collection or not.
2. In the roster method (tabular form), the members (elements) of the set are listed; while in the rule method (set-builder form), the members are described by a common property of the members.
3. In listing the members of a set, identical (same) members are included only once. The members can be listed in any order.
4.

Symbol	Meaning
$x \in A$	x is a member of the set A.
$x \notin A$	x is not a member of the set A.
$A = B$	A and B are equal sets.
$n(A)$	the number of distinct members of A.
$A \leftrightarrow B$	A and B are equivalent sets; that is, $n(A) = n(B)$.

5. The set A is
   (i) a finite set if $n(A)$ is finite
   (ii) an infinite set if $n(A)$ is not finite
   (iii) a singleton if $n(A) = 1$
   (iv) a pair set if $n(A) = 2$
   (v) the empty set, i.e. $\emptyset$, if $n(A) = 0$.
6. *Some special sets*
   (i) $N = \{1, 2, 3, 4, \ldots\}$ is the set of natural numbers.
   (ii) $W = \{0, 1, 2, 3, \ldots\}$ is the set of whole numbers.
   (iii) I or $Z = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ is the set of integers.
   (iv) $\emptyset$ is the empty set. $n(\emptyset) = 0$.
   (v) U or ξ is a universal set. It may be finite or infinite.

EXERCISE 1A

1. Which of the following collections are sets?
   (i) All the months of a year
   (ii) All superactors of India
   (iii) All the letters of the word MIZORAM
   (iv) All the natural numbers less than 20 which are perfect squares
   (v) All the integers lying between −5 and 5
   (vi) All the good boys of our neighbourhood
2. Let A be the set of all whole numbers greater than −3 but less than 5. Then, fill in the blanks by using $\in$ or $\notin$.
   (i) 0 ...... A (ii) 3 ...... A (iii) −4 ...... A (iv) 5 ...... A

3. Write the following sets by the roster as well as the rule method.

(i) The set of letters of the word TERRACOTTA

(ii) The set of whole numbers lying between −4 and 2

(iii) The set of consonants in the word RABINDRANATH

(iv) The set of even integers lying between −10 and 10

4. Write the following sets in the tabular form and find their cardinal numbers.

(i) $A = \{x | x \in N, -1 < x < 3\}$

(ii) $B = \{x | x$ is a prime number of one digit$\}$

(iii) $C = \{x | x$ is a two-digit number divisible by 15$\}$

(iv) $D = \{x | x$ is a letter of the word BHARATI$\}$

5. Write the following sets in the set-builder form.

(i) {a, e, i, o, u} (ii) {0, 5, 10, 15}

(iii) {1, 3, 5, 7, ...} (iv) {..., −3, −2, −1, 0}

6. Identify the finite and infinite sets, and find the cardinal numbers of the finite sets.

(i) $A = \{x | x > 4 \text{ and } x \in N\}$ (ii) $B = \{x | x < 5 \text{ and } x \in W\}$

(iii) $C = \{x | x < 5 \text{ and } x \in Z\}$ (iv) $D = \{..., -3, -2, -1, 0\}$

(v) $E = \{x | x < 1 \text{ and } x \in N\}$ (vi) $F = \left\{x \middle| x = \frac{1}{n^2+1}, n \in Z\right\}$

7. Identify the empty set, the singleton and the pair set.

(i) $A = \{0\}$ (ii) $B = \{x | x^2 - 9 = 0, x \in Z\}$

(iii) $C = \{x | x^2 = 25, x \in N\}$ (iv) $D = \{x | 5x - 4 = 0, x \in W\}$

8. Let $X = \{x | x$ is a letter of the word MINISTER$\}$ and $Y = \{x | x$ is a letter of the word SINISTER$\}$. State which of the following are true and which are false.

(i) $X = Y$ (ii) $X \leftrightarrow Y$

9. Let $P = \{1, -1\}$, $Q = \{x | x^2 = 4, x \in Z\}$ and $R = \{-2, -1, 1\}$. State which of the following are true and which are false.

(i) $P \leftrightarrow Q$ (ii) $P = Q$ (iii) $Q \leftrightarrow R$ (iv) $P \leftrightarrow R$

(v) $Q = R$ (vi) $P = R$

10. Let $A = \{1, 2, 3, 4\}$, $B = \{a, b, c, d\}$ and $C = \{x | x < 5, x \in N\}$. State which of the following are true and which are false.

(i) $A \leftrightarrow B$ (ii) $A = B$ (iii) $A = C$ (iv) $B \leftrightarrow C$

(v) $B = C$

11. Let $A = \{1, 3, 5, 7, ...\}$, $B = \{x | x = 2n - 1, n \in N\}$ and $C = \{x | x = 4n - 1, n \in N\}$. Prove that

(i) $A = B$ (ii) $C \neq B$.

12. Write the set $A = \{x | 3x + 1 < 15\}$ in the tabular form when the universal set is

(i) N (ii) W (iii) Z.

13. Write the set $A = \{x | x^2 < 4\}$ by the roster method when the universal set is

(i) N (ii) W (iii) Z.

ANSWERS

1. (i), (iii), (iv) and (v)

2. (i) $\in$ (ii) $\in$ (iii) $\notin$ (iv) $\notin$

3. (i) {T, E, R, A, C, O} = {x | x is a letter of the word TERRACOTTA}
(ii) {0, 1} = {x | $x \in W$, $-4 < x < 2$}
(iii) {R, B, N, D, T, H} = {x | x is a consonant in the word RABINDRANATH}
(iv) {−8, −6, −4, −2, 0, 2, 4, 6, 8} = {x | $x = 2n$, $n \in Z$, $-4 \le n \le 4$}

4. (i) A = {1, 2} and $n(A) = 2$ (ii) B = {2, 3, 5, 7} and $n(B) = 4$
(iii) C = {15, 30, 45, 60, 75, 90} and $n(C) = 6$ (iv) D = {B, H, A, R, T, I} and $n(D) = 6$

5. (i) {x | x is a vowel of the English alphabet} (ii) {x | $x = 5n$, $n \in W$ and $x \le 15$}
(iii) {x | x is an odd natural number} (iv) {x | $x \in Z$, $x \le 0$}

6. (i) Infinite set (ii) Finite set, where $n(B) = 5$ (iii) Infinite set (iv) Infinite set
(v) Finite set, where $n(E) = 0$ (vi) Infinite set

7. (i) Singleton (ii) Pair set (iii) Singleton (iv) Empty set

8. (i) False (ii) False

9. (i) True (ii) False (iii) False (iv) False (v) False (vi) False

10. (i) True (ii) False (iii) True (iv) True (v) False

12. (i) {1, 2, 3, 4} (ii) {0, 1, 2, 3, 4} (iii) {…, −3, −2, −1, 0, 1, 2, 3, 4}

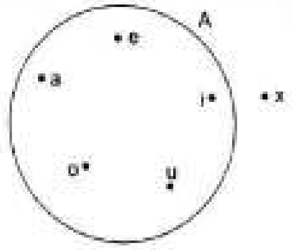
13. (i) {1} (ii) {0, 1} (iii) {−1, 0, 1}

Venn Diagrams

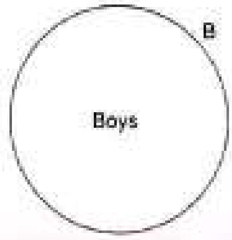
A diagram used to represent a set is called a Venn diagram. Such a diagram can also be used to display a combination of two or more sets.

Representation of a set

A set is usually represented by a circle or an oval-shaped closed curve. The members of the set are represented by points inside the figure and the name of the set is written on the boundary of the figure. Objects that do not belong to the set are shown as points outside the figure. The adjoining figure represents the set A = {a, e, i, o, u}. Any object x which does not belong to A is represented by a point outside the figure.



If a set has a large number of members then, instead of representing the members by points, we write the description of the set inside the circle. To display the set B of boys of your class, you can draw the diagram shown alongside.

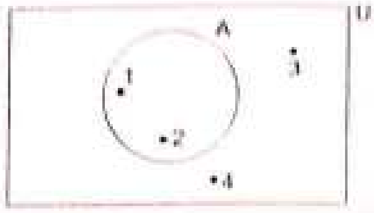


Representation of a set along with the universal set

The universal set is usually represented by a rectangle to distinguish it from the other sets. In the Venn diagram given alongside, the circle representing the set A is drawn inside the rectangle showing the universal set U because all the members of the set A are also members of U.

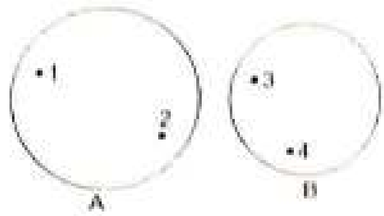
All the elements of a set are shown inside the circle. An object which is not a member of the set is shown outside the circle but inside the rectangle.

Example The universal set $U = \{1, 2, 3, 4\}$ and the set $A = \{1, 2\}$ are represented by the adjoining diagram.



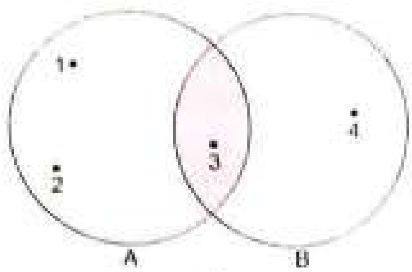
Representation of two sets by a diagram

Let $A = \{1, 2\}$ and $B = \{3, 4\}$. This is shown by the Venn diagram (i). Here, the members 1 and 2 of the set A are shown by the points inside the circle representing A. The members 3 and 4 of the set B are shown inside the circle representing B.



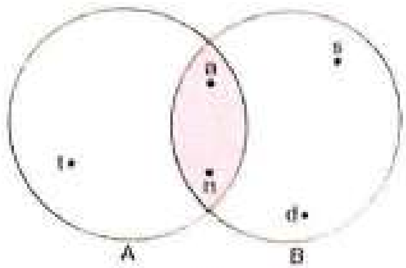
(i)

Let $A = \{1, 2, 3\}$ and $B = \{3, 4\}$. This is shown by the adjoining Venn diagram (ii). Here, the member 3 belongs to both the sets A and B. So, the member 3 is shown in the common portion of both the circles representing the sets A and B.



(ii)

Let $A = \{a, n, t\}$ and $B = \{s, a, n, d\}$. This is shown by the adjoining Venn diagram (iii). Here, the members a and n belong to both the sets A and B, and are shown in the common portion of both the circles representing A and B.



(iii)

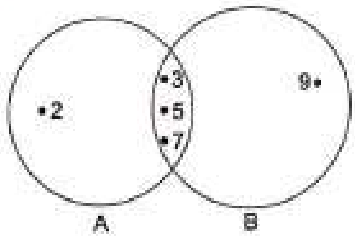
When two sets have no members in common, they are called **disjoint sets**. The sets A and B in the figure (i) are disjoint sets.

If two sets have at least one common member, they are called **overlapping sets**. The sets A and B in the figures (ii) and (iii) are overlapping sets.

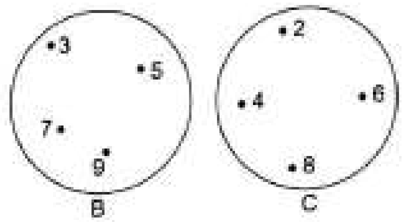
EXAMPLE **Let $A = \{x \mid x$ is a prime factor of $210\}$, $B = \{3, 5, 7, 9\}$ and $C = \{2, 4, 6, 8\}$. Verify that (i) A and B are overlapping sets, (ii) B and C are disjoint sets, and (iii) A and C are overlapping sets. Also, show them by Venn diagrams.**

Solution Writing in the tabular form, $A = \{2, 3, 5, 7\}$.

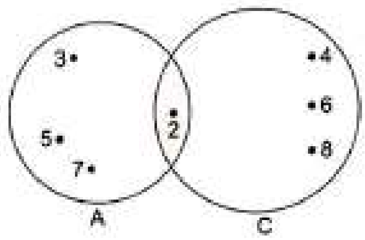
(i) We find that the members 3, 5 and 7 are common to A and B. So, A and B are overlapping sets.



(ii) No members are common to B and C. So, B and C are disjoint sets.



(iii) Since $2 \in A$ and $2 \in C$, A and C are overlapping sets.



Solved Examples

EXAMPLE 1 **Answer the following from the adjoining Venn diagram.**

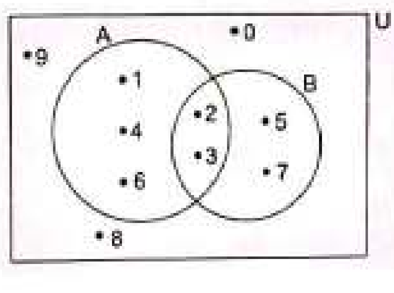
(a) Fill in by using $\in$ or $\notin$.

(i) 2...... A **(ii) 3...... B** **(iii) 5...... A**

(iv) 8...... A **(v) 9...... B** **(vi) 6...... B**

(b) Write the sets in the tabular form.

(i) A **(ii) B** **(iii) U**



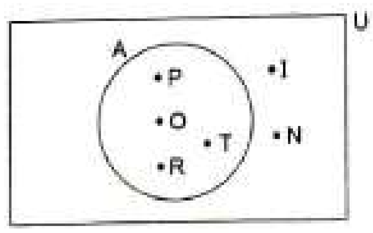
Solution (a) (i) $\in$ (ii) $\in$ (iii) $\notin$ (iv) $\notin$ (v) $\notin$ (vi) $\notin$

(b) (i) $A = \{1, 4, 6, 2, 3\}$ (ii) $B = \{2, 3, 5, 7\}$ (iii) $U = \{1, 4, 6, 2, 3, 5, 7, 0, 8, 9\}$

EXAMPLE 2 **Draw a Venn diagram to represent $U = \{x \mid x$ is a letter of the word PROPORTION$\}$ and $A = \{P, O, R, T\}$.**

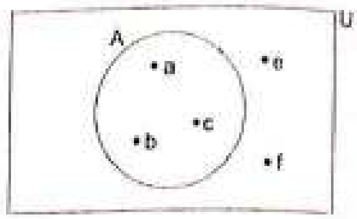
Solution Here, $U = \{P, R, O, T, I, N\}$ and $A = \{P, O, R, T\}$.

$\therefore$ the required Venn diagram is as given alongside.

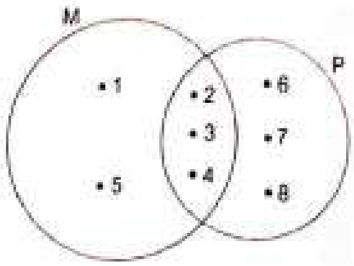


EXERCISE 1B

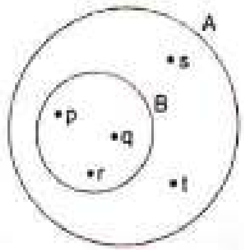
1. Answer the following from the adjoining Venn diagram.
   (i) Write the set A by the roster method.
   (ii) Write the universal set U in the tabular form.
   (iii) Fill in the blanks using $\in$ or $\notin$.
   (a) a ...... A (b) e ...... A (c) f ...... A'



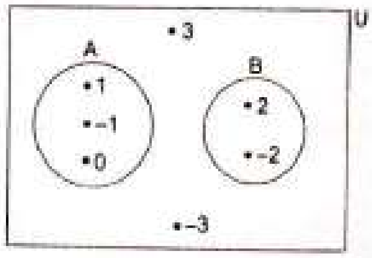
2. Answer the following from the adjoining Venn diagram.
   (i) Write the sets M and P in the tabular form.
   (ii) Write the members common to M and P.



3. Answer the following from the adjoining Venn diagram.
   (i) Write the sets A and B in the tabular form.
   (ii) Write the members common to A and B.



4. Answer the following from the adjoining Venn diagram.
   (i) Write the universal set U by the roster method.
   (ii) Write the sets A and B in the tabular form.
   (iii) Are A and B disjoint sets?

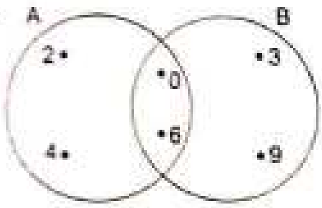


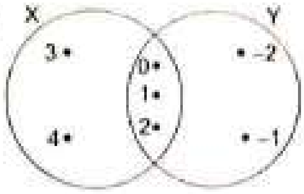
5. If $A = \{0, 2, 4, 6\}$ and $B = \{0, 3, 6, 9\}$ then represent them by a Venn diagram.
6. If $X = \{x | x \in W, x < 5\}$ and $Y = \{x | x \in Z, -2 \le x \le 2\}$ then represent them by a Venn diagram.
7. Represent the universal set $U = \{1, 2, 3, 4, 5, 6, 7\}$ and $A = \{1, 3, 5, 7\}$ by a Venn diagram.
8. Let $X = \{2, 4, 6\}$, $Y = \{3, 6, 9\}$ and $Z = \{4, 8, 12, 16\}$. Identify the disjoint and overlapping sets, among (i) X and Y, (ii) Y and Z, and (iii) Z and X. Also, show them by Venn diagrams.
9. Let $A = \{x | x \in W, x$ is a multiple of 5 and $x < 30\}$, $B = \{x | x \in N, x$ is divisible by 7 and $x < 30\}$ and $C = \{x | x \in W$ and $x < 5\}$. Verify that (i) A and B are disjoint sets, (ii) A and C are overlapping sets, and (iii) B and C are nonoverlapping sets.
10. Represent each of the following by a Venn diagram.
    (i) $A = \{1, 2\}$ and $B = \{-1, -2\}$
    (ii) $X = \{a, l, k\}$ and $Y = \{i, g, l\}$
    (iii) $M = \{c, a, k, e\}$ and $N = \{d, a, r, k\}$

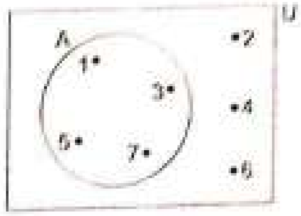
11. Draw a Venn diagram to represent the sets $U = \{x \mid x$ is a letter of the word ENTERTAINMENT$\}$ and $X = \{A, I, R, T\}$.

ANSWERS

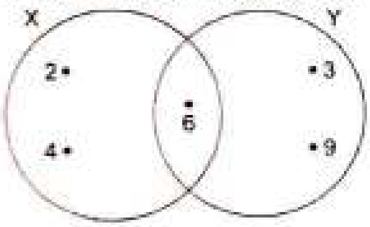
1. (i) A = {a, b, c} (ii) U = {a, b, c, e, i} (iii) (a) ∈ (b) ∉ (c) ∈
2. (i) M = {1, 2, 3, 4, 5}, P = {2, 3, 4, 6, 7, 8} (ii) 2, 3 and 4
3. (i) A = {p, q, r, s, t}, B = {p, q, r} (ii) p, q and r
4. (i) U = {0, −1, 1, 2, −2, 3, −3} (ii) A = {1, −1, 0} and B = {2, −2} (iii) yes

5. 

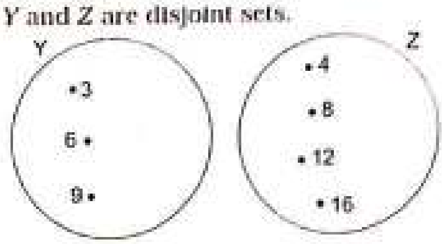
6. 

7. 

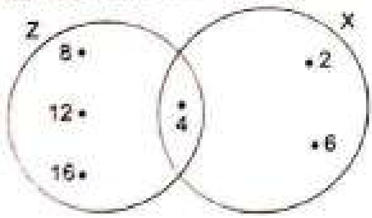
8. (i) X and Y are overlapping sets.

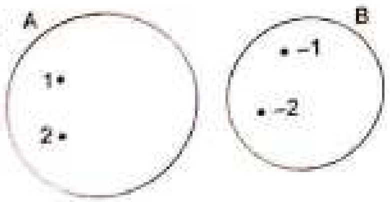


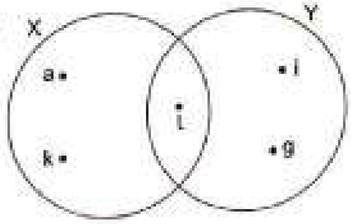
(ii) Y and Z are disjoint sets.

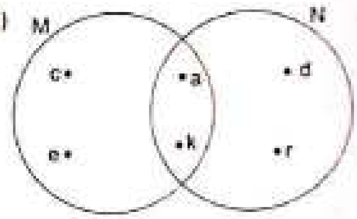


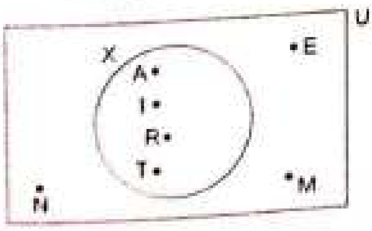
(iii) Z and X are overlapping sets.



10. (i) 

(ii) 

(iii) 

11. 

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Revision Exercise

1. Which of the following collections is a set?

(i) Four consecutive integers

(ii) All prime numbers lying between 4 and 12

2. Write the set $A = \{x \mid x = n^2 + 1, n \in W \text{ and } x < 15\}$ by the roster method and fill in the blanks by using $\in$ or $\notin$.

(i) 11 ...... A (ii) 5 ...... A (iii) 17 ...... A (iv) 10 ...... A

3. Write the set P of natural numbers less than 11 in the tabular as well as the set-builder form. Is P a finite set? If so, find $n(P)$.

4. Classify the following sets into finite, infinite and empty sets.

(i) The set of prime numbers lying between 13 and 17

(ii) The set of natural numbers which are multiples of 11

(iii) $P = \{x \mid x$ is a letter of the word EXAMINATION$\}$

(iv) $A = \{x \mid x \in W, 4x - 3 < 0\}$

5. Let $A = \{x \mid W, x^2 < 9\}$, $B = \{x \mid x \in N, 2x < 9\}$, $C = \{1, 2\}$ and $D = \{3, 4, 5, 6\}$. Which of the following are true and which are false?

(i) $A = D$ (ii) $B = A$ (iii) $B \leftrightarrow D$ (iv) $C = D$

(v) $B = C$ (vi) $C = A$ (vii) $A \leftrightarrow B$

6. If $A = \{x \mid x \in N, 2x < 11\}$ and $B = \{x \mid x$ is an odd natural number and $x < 10\}$, write (i) A and B in the tabular form, (ii) find $n(A)$, (iii) find $n(B)$, and (iv) find the common members of A and B.

7. Let the universal set be $U = \{x \mid x \in W, x + 1 \leq 10\}$. If $A = \{1, 3, 9\}$ then show them by a Venn diagram.

8. Let $A = \{a, b, c, m, n\}$, $B = \{c, d, e, f\}$ and $C = \{m, n, p\}$. Verify that (i) A and B are overlapping sets, (ii) B and C are disjoint sets, and (iii) A and C are overlapping sets.

9. Let the universal set be $U = \{x \mid x \in N, x < 10\}$, and also let $A = \{1, 3, 5\}$ and $B = \{2, 4, 6, 8\}$. Show them by a Venn diagram.

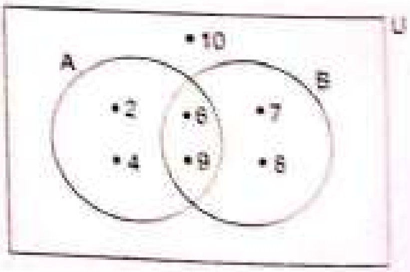
10. Answer the following from the adjoining Venn diagram.

(i) Write the sets A and B in the tabular form.

(ii) Find the members common to A and B.

(iii) Find the universal set U.

(iv) Verify if $A \leftrightarrow B$.



11. If $U = \{x \mid x \in N, 1 \leq x \leq 10\}$, $A = \{2, 4, 6, 8, 10\}$ and $B = \{4, 6, 8\}$, draw a Venn diagram to represent them.

ANSWERS

1. (ii)

2. $\{1, 2, 5, 10\}$; (i) $\notin$ (ii) $\in$ (iii) $\notin$ (iv) $\in$

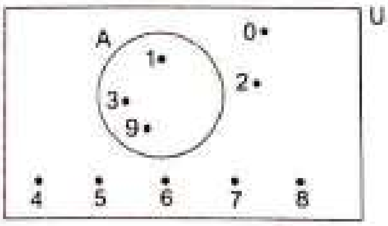
3. $P = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} = \{x \mid x \in N, x < 11\}$; P is a finite set; $n(P) = 10$

4. (i) Empty set (ii) Infinite set (iii) Finite set (iv) Finite set

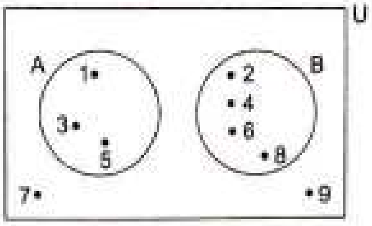
5. (i) False (ii) False (iii) True (iv) False (v) False (vi) False (vii) False

6. (i) $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 3, 5, 7, 9\}$ (ii) 5 (iii) 5 (iv) 1, 3 and 5

7.

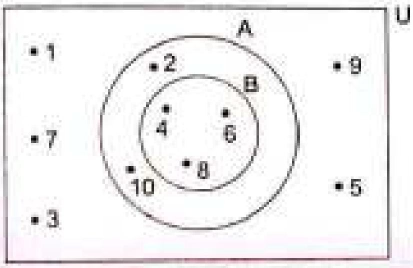


9.



10. (i) $A = \{2, 4, 6, 9\}$ and $B = \{6, 7, 8, 9\}$ (ii) 6 and 9 (iii) $\{2, 4, 6, 7, 8, 9, 10\}$ (iv) Yes

11.



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1

Integers

You already know that for each natural number a, there is a number $-a$ such that $a+(-a)=0$. Thus, $1+(-1)=0,\ 2+(-2)=0,\ 3+(-3)=0, \ldots$.

The numbers ..., –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, ... are called integers. 1, 2, 3, 4, 5, ... are called positive integers, while –1, –2, –3 –4, –5, ... are called negative integers. The integer 0 (zero) is neither positive nor negative.

The set of integers is denoted by I or Z. It is clear that all the natural numbers and all the whole numbers are integers. Hence, if $x \in N$ then $x \in W$ as well as $x \in I$.

Representation of integers on the number line

Negative integers are marked on the left side of zero on the number line, while positive integers are shown on the right.

The distance of a negative integer from zero is the same as the distance of the corresponding positive integer from zero. For example, the distance between –2 and zero is the same as the distance between 2 and zero.

An integer is greater than another integer if it lies to the right of the other integer on the number line. For example, –3 lies to the right of –6. Therefore, $-3 > -6$. Similarly, an integer is smaller than another integer if it lies to the left of the other integer on the number line. For example, –5 lies to the left of –4. Therefore, $-5 < -4$.

The following should be clear from the number line.

1. There is no greatest or smallest integer.
2. Every integer has a successor and a predecessor. For an integer n, the predecessor is $n-1$, while the successor is $n+1$.

Absolute value of an integer

The absolute value of an integer is the whole number obtained by disregarding its sign. The absolute value of an integer a is denoted by $|a|$.

Thus, $|-7|$ = (absolute value of –7) = 7,

$|+5|$ = (absolute value of +5) = 5,

and $|0|$ = (absolute value of 0) = 0.

Comparison of integers

Remember the following while comparing integers.

1. All positive integers are greater than zero and all negative integers are less than zero.
2. Every positive integer is greater than all negative integers.
3. Given two positive integers, the one with the bigger absolute value is greater
4. Given two negative integers, the one with the smaller absolute value is greater.

Solved Examples

EXAMPLE 1 **Write the following sets of numbers in the set-builder and tabular forms, and represent the numbers on the number line.**

(i) Integers greater than 3 and less than 7

(ii) Integers greater than or equal to −2 and less than 3

(iii) Integers greater than or equal to −3 and less than or equal to 3

(iv) Integers greater than −5 and less than or equal to 0

Solution (i) Set-builder form: $\{x \mid x \in I, 3 < x < 7\}$; tabular form: $\{4, 5, 6\}$.

(ii) Set-builder form: $\{x \mid x \in I, -2 \le x < 3\}$; tabular form: $\{-2, -1, 0, 1, 2\}$.

(iii) Set-builder form: $\{x \mid x \in I, -3 \le x \le 3\}$; tabular form: $\{-3, -2, -1, 0, 1, 2, 3\}$.

(iv) Set-builder form: $\{x \mid x \in I, -5 < x \le 0\}$; tabular form: $\{-4, -3, -2, -1, 0\}$.

EXAMPLE 2 **Find the following.**

(i) $|-3| - |9|$ **(ii) $-|-26| + |-25| - |39|$** **(iii) $|36| + |14| - |-50| + |20|$**

Solution (i) $|-3| - |9| = 3 - 9 = -6$.

(ii) $-|-26| + |-25| - |39| = -26 + 25 - 39 = 25 - 65 = -40$.

(iii) $|36| + |14| - |-50| + |20| = 36 + 14 - 50 + 20 = 70 - 50 = 20$.

Remember These

1. I (or Z) = set of integers = $\{\ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots\}$.

2. The absolute value of an integer is the number obtained by omitting its sign.

3. (i) Given two positive integers, the number with the greater absolute value is bigger.
(ii) Given two negative integers, the number with the smaller absolute value is bigger.
(iii) Each positive integer is greater than all negative integers.

EXERCISE 1A

1. Write the following sets of numbers in the set-builder and tabular forms, and represent them on the number line.
   (i) Integers greater than –4 and less than 1
   (ii) Integers greater than or equal to 0 and less than 4
   (iii) Integers greater than or equal to –2 and less than or equal to 0
   (iv) Integers greater than –1 and less than or equal to 4
2. Fill in the blanks by using >, < or =.
   (i) –3 ...... 1 (ii) –8 ...... –4 (iii) 2 ...... –5 (iv) –12 ...... –18
   (v) $|-5|$ ...... 5 (vi) $|-15|$ ...... $|17|$ (vii) $|-17| - |15|$ ...... $|(-17) + 15|$
3. Arrange the following integers in the ascending order.
   (i) –37, 132, –45, –172, 33, 35, 0
   (ii) –172, –247, 136, 242, –365, –473, –36, 0, 77
4. Find the following.
   (i) $|-3| - |6-2|$ (ii) $|-84| + |-16|$ (iii) $|-12| + |6| - |-3|$
   (iv) $|38| - |126 - 167|$ (v) $|273| - |-343| - |12| + |-2|$
5. Indicate whether the following are true or false.
   (i) The largest integer is 9999. (ii) The smallest integer is zero.
   (iii) The absolute value of an integer is always greater than the integer.
   (iv) The successor of 125 is 124. (v) The predecessor of –123 is –124.

ANSWERS

1. (i) $\{x \mid -4 < x < 1, x \in I\}$ and {–3, –2, –1, 0}:


(ii) $\{x \mid 0 \le x < 4, x \in I\}$ and {0, 1, 2, 3}:


(iii) $\{x \mid -2 \le x \le 0, x \in I\}$ and {–2, –1, 0}:


(iv) $\{x \mid -1 < x \le 4, x \in I\}$ and {0, 1, 2, 3, 4}:


2. (i) < (ii) < (iii) > (iv) > (v) = (vi) < (vii) =
3. (i) –172, –45, –37, 0, 33, 35, 132 (ii) –473, –365, –247, –172, –36, 0, 77, 136, 242
4. (i) –1 (ii) 100 (iii) 15 (iv) –3 (v) –80
5. (i) False (ii) False (iii) False (iv) False (v) True

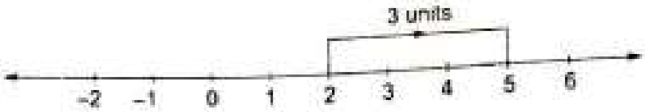
Addition and Subtraction of Integers

Addition of integers using the number line

To add a positive integer to a given integer, move to the right of the given integer by as many unit distances as the absolute value of the integer to be added.

EXAMPLE **Add +3 to +2.**

Solution From +2 we move 3 units to the right and arrive at 5.

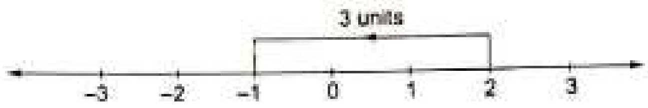


$\therefore$ (+3) + (+2) = +5 = 5.

To add a negative integer to a given integer, move to the left of the given integer by as many unit distances as the absolute value of the integer to be added.

EXAMPLE **Add –3 to +2.**

Solution From +2, we move 3 units to the left and arrive at –1.



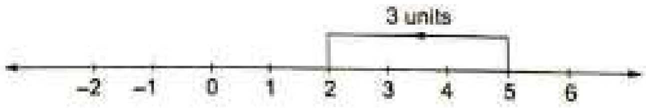
$\therefore$ (–3) + (+2) = –1.

Subtraction of integers using the number line

To subtract a positive integer, move to the left of the given integer by as many unit distances as the absolute value of the integer to be subtracted.

EXAMPLE **Subtract +3 from +5.**

Solution From +5, we move 3 units to the left and arrive at +2.

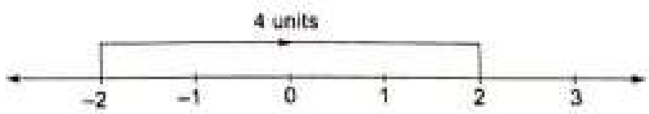


$\therefore$ (+5) – (+3) = +2 or 5 – 3 = 2.

To subtract a negative integer, move to the right of the given integer by as many unit distances as the absolute value of the integer to be subtracted.

EXAMPLE **Subtract –4 from –2.**

Solution From –2, we move 4 units to the right and arrive at 2.



$\therefore$ (–2) – (–4) = +2.

Remember These

1. To add a positive integer +a, move a units to the right of the given integer.
2. To add a negative integer −a, move a units to the left of the given integer.
3. Subtraction is the opposite of addition.

EXERCISE 1B

1. Add the following numbers using the number line.
   (i) (+6) + (+2) (ii) (−2) + (+4) (iii) (+7) + (−3) (iv) (−3) + (−3)
2. Use the number line to find the following.
   (i) 3 − 5 (ii) (−2) − (−3) (iii) (−3) − 4 (iv) 5 − (−3)

ANSWERS

1. (i) 8 (ii) 2 (iii) 4 (iv) −6 **2.** (i) −2 (ii) 1 (iii) −7 (iv) 8

Addition of integers

Addition of two integers of the same sign

To add two integers of the same sign (i.e., both positive or both negative), add their absolute values and attach the common sign to the sum.

EXAMPLE **Add +256 and +34.**

Solution $|+256| = 256$ and $|+34| = 34$.

$\therefore \quad (+256) + (+34) = 256 + 34 = 290.$

EXAMPLE **Add −112 and −240.**

Solution $|-112| + |-240| = 112 + 240 = 352.$

$\therefore \quad (-112) + (-240) = -(112 + 240) = -352.$

Addition of two integers of opposite signs

To add two integers of opposite signs, take the following steps.

Steps
1. Find the absolute values of the integers.
2. Subtract the smaller absolute value from the bigger absolute value.
3. To the difference in Step 2, attach the sign of the integer which has the larger absolute value.

EXAMPLE **Add −33 and +24.**

Solution $|-33| = 33$, $|+24| = 24$; $33 - 24 = 9$.
−33 has the greater absolute value; so $(-33) + (+24) = -9$.

EXAMPLE **Add +136 and −92.**

Solution $|+136| = 136$, $|-92| = 92$; $136 - 92 = 44$.
+136 has the greater absolute value; so $(+136) + (-92) = 44$.

Properties of addition

1. The addition of integers is commutative; i.e., if a and b are any two integers,

$$a + b = b + a.$$

Examples (i) $26 + 37 = 37 + 26$. (ii) $(-12) + 26 = 26 + (-12)$.
(iii) $(-126) + (-284) = (-284) + (-126)$. (iv) $45 + (-94) = (-94) + 45$.

2. The addition of integers is associative; i.e., if a, b and c are any three integers,

$$(a + b) + c = a + (b + c) = a + b + c.$$

Examples (i) $\{67 + (-132)\} + 284 = 67 + \{(-132) + 284\}$.
(ii) $(396 + 467) + (-223) = 396 + \{467 + (-223)\}$.

3. The sum of any integer and zero is the integer itself. In other words, if a is an integer, $a + 0 = a$. Zero is known as the additive identity.

EXAMPLE **Add −26, 98, 22 and −34.**

Solution $-26 + 98 + 22 + (-34) = \{-26 + (-34)\} + (98 + 22) = -(26 + 34) + 120 = -60 + 120 = 60$.

Subtraction of integers

Subtraction is the opposite of addition. Thus, to subtract an integer from a given integer, add the integer of the opposite sign to the given integer. Symbolically,

$$a - b = a + (-b).$$

Examples (i) $19 - 11 = 19 + (-11) = +8 = 8$.
(ii) $115 - 143 = 115 + (-143) = -(143 - 115) = -28$.
(iii) $(-16) - (11) = (-16) + (-11) = -(16 + 11) = -27$.

Solved Examples

EXAMPLE 1 **Find the following.**

(i) $-12 - (-68)$ (ii) $237 - (-576)$ (iii) $-(-786) + (-2420)$ (iv) $(-4860) - (-6247)$

Solution (i) $-12 - (-68) = -12 + 68 = 56$. (ii) $237 - (-576) = 237 + 576 = 813$.
(iii) $-(-786) + (-2420) = 786 - 2420 = -1634$.
(iv) $(-4860) + (+6247) = -4860 + 6247 = 1387$.

EXAMPLE 2 **Find the following.**

(i) $12 + (-14) + 16 - 18 + 19 - 45$ (ii) $-24 + 36 - 145 + (-679) + 285 - (+386)$

Solution (i) $12 + (-14) + 16 - 18 + 19 - 45 = (12 + 16 + 19) - (14 + 18 + 45) = 47 - 77 = -30.$

(ii) $(-24) + 36 - 145 + (-679) + 285 - (+386) = (-24 - 145 - 679 - 386) + (36 + 285)$

$= -1234 + 321 = -913.$

Remember These

1. To add two positive integers, add their absolute values and attach the plus (+) sign.
2. To add two negative integers, add their absolute values and attach the negative (–) sign.
3. To add two integers of opposite signs, find the difference of their absolute values and attach the sign of the integer having the greater absolute value.
4. To subtract an integer from a given integer, add the integer of the opposite sign to the given integer.

EXERCISE 1C

1. Add the following.

(i) $(+212) + (+67)$ (ii) $(+678) + (+1321)$

(iii) $(-120) + (-269)$ (iv) $(-367) + (-4293)$

(v) $169 + (-125)$ (vi) $-940 + 772$

(vii) $316 + (-472) + 735$ (viii) $-2167 + 179 + (-4201) + 812$

(ix) $(+1234) + 762 + (-2467) + 3468 + (-9802)$

2. Evaluate the following.

(i) $(-12) - (-63)$ (ii) $-125 - (+176)$

(iii) $12 - 24 + 7 - 9$ (iv) $28 - (-43) + (-16) - (-128) - 68$

(v) $64 - 48 + 151 - 123 + (-124) - (-167)$

ANSWERS

1. (i) 279 (ii) 1999 (iii) –389 (iv) –4660 (v) 44 (vi) –168 (vii) 579 (viii) –5377 (ix) –6805

2. (i) 51 (ii) –301 (iii) –14 (iv) 115 (v) 87

Multiplication and Division of Integers

Multiplication of integers

Multiplication of two integers of the same sign

The product of two integers of the same sign is a positive integer equal to the product of the absolute values of the integers.

Examples (i) $(+16)\times(+8) = +(16\times 8) = +128 = 128$.

(ii) $(-24)\times(-30) = +(|-24|\times|-30|) = +(24\times 30) = +720 = 720$.

Multiplication of two integers of opposite signs

The product of two integers of opposite signs is a negative integer whose absolute value is equal to the product of their absolute values.

Examples (i) $(-24)\times(+25) = -(|-24|\times|+25|) = -(24\times 25) = -600$.

(ii) $(+125)\times(-36) = -(|+125|\times|-36|) = -(125\times 36) = -4500$.

Properties of multiplication

1. The multiplication of integers is commutative; i.e., if a and b are any two integers,

$$a\times b = b\times a.$$

Examples (i) $24\times 28 = 28\times 24$. (ii) $(-23)\times 73 = 73\times(-23)$.

(iii) $(-26)\times(-123) = (-123)\times(-26)$. (iv) $375\times(-497) = (-497)\times 375$.

2. The multiplication of integers is associative; i.e., if a, b and c are any three integers,

$$(a\times b)\times c = a\times(b\times c) = a\times b\times c.$$

Examples (i) $(24\times 72)\times 28 = 24\times(72\times 28)$.

(ii) $\{237\times(-676)\}\times 125 = 237\times\{(-676)\times 125\}$.

3. The product of any integer and zero is zero. Symbolically,

$$a\times 0 = 0\times a = 0.$$

Examples (i) $4\times 0 = 0\times 4 = 0$. (ii) $(-27)\times 0 = 0\times(-27) = 0$.

4. The product of any nonzero integer and 1 is the integer itself. 1 is called the mutiplicative identity. Symbolically,

$$a\times 1 = a.$$

5. Multiplication is distributive over addition. In other words, if a, b and c are any three integers then

$$a \times (b + c) = a \times b + a \times c.$$

Examples (i) $24 \times (32 + 46) = 24 \times 32 + 24 \times 46$.

(ii) $(78 + 927) \times 35 = 78 \times 35 + 927 \times 35$.

Division of integers

Division of an integer by an integer of the same sign

If an integer is divided by another integer of the same sign then the quotient is a positive integer whose absolute value is found by dividing the absolute values of the integers.

Examples (i) $(+48) \div (+8) = |+48| \div |+8| = 48 \div 8 = 6$.

(ii) $(-75) \div (-15) = |-75| \div |-15| = 75 \div 15 = 5$.

Division of an integer by an integer of the opposite sign

If an integer is divided by an integer of the opposite sign then the quotient is a negative integer whose absolute value is found by dividing the absolute values of the integers.

Examples (i) $(+90) \div (-15) = -(|+90| \div |-15|) = -(90 \div 15) = -6$.

(ii) $(-115) \div (+23) = -(|-115| \div |+23|) = -(115 \div 23) = -5$.

Notes
- If a is any integer then $a \div 1 = a$ and $a \div (-1) = -a$.
- If a be any nonzero integer then $0 \div a = 0$.
- Division of an integer by zero is not permitted.

Simplification

An expression involving integers is simplified by the rule of BODMAS.

Steps 1. First, carry out the operations inside the brackets (B).

Brackets are removed in this order: line bracket (or vinculum), first (or round) brackets, second (or curly) brackets, third (or rectangular or square) brackets.

2. Then change 'of' into '×' and multiply (O).

3. Carry out the operations of division and multiplication, in the same order as in the given expression (DM).

4. Finally, carry out the operations of addition and subtraction (AS).

EXAMPLE **Simplify.** $52 - 60 \div [3 \text{ of } \{2 \times (8 \div 3 + 1) + 6\}]$

Solution The given expression $= 52 - 60 \div [3 \text{ of } \{2 \times (8 \div 4) + 6\}] = 52 - 60 \div [3 \text{ of } \{2 \times 2 + 6\}]$

$= 52 - 60 \div [3 \text{ of } \{4 + 6\}] = 52 - 60 \div [3 \text{ of } 10]$

$= 52 - 60 \div [3 \times 10] = 52 - 60 \div 30 = 52 - 2 = 50.$

Solved Examples

EXAMPLE 1 **Find the following products.**

(i) $8 \times (+25)$ (ii) $(-14) \times (-35)$ (iii) $(-25) \times 35$ (iv) $(+12) \times (-32)$

Solution

(i) $8 \times (+25) = +(8 \times 25) = 200.$

(ii) $(-14) \times (-35) = +(14 \times 35) = 490.$

(iii) $(-25) \times 35 = -(25 \times 35) = -875.$

(iv) $(+12) \times (-32) = -(12 \times 32) = -384.$

EXAMPLE 2 **Find the following products.**

(i) $6 \times (-42) \times (-27)$ (ii) $(-12) \times (-27) \times (-35) \times 8$ (iii) $26 \times 4 \times 37 \times (-25)$

(iv) $4 \times (-6) \times (-5) \times (-3) \times 5$ (v) $8 \times 12 \times (-15) \times 20 \times 25 \times 37$

Solution

(i) $6 \times (-42) \times (-27) = \{6 \times (-42)\} \times (-27) = (-252) \times (-27) = 6804.$

(ii) $(-12) \times (-27) \times (-35) \times 8 = \{(-12) \times (-27)\} \times \{(-35) \times 8\} = 324 \times (-280) = -90720.$

(iii) $26 \times 4 \times 37 \times (-25) = (26 \times 37) \times \{4 \times (-25)\} = 962 \times (-100) = -96200.$

(iv) $4 \times (-6) \times (-5) \times (-3) \times 5 = \{4 \times (-5) \times 5\} \times \{(-6) \times (-3)\} = -100 \times 18 = -1800.$

(v) $8 \times 12 \times (-15) \times 20 \times 25 \times 37 = (8 \times 12) \times \{(-15) \times 20\} \times (25 \times 37)$

$= 96 \times (-300) \times 925 = -(96 \times 300) \times 925$

$= -(28800 \times 925) = -26640000.$

EXAMPLE 3 **Find the quotient.**

(i) $(+125) \div (+25)$ (ii) $(-323) \div 17$ (iii) $483 \div (-21)$ (iv) $(-255) \div (-15)$

Solution

(i) $(+125) \div (+25) = +(125 \div 25) = 5.$

(ii) $(-323) \div 17 = -(323 \div 17) = -19.$

(iii) $483 \div (-21) = -(483 \div 21) = -23.$

(iv) $(-255) \div (-15) = +(255 \div 15) = 17.$

EXAMPLE 4 **Simplify.** (i) $25 - \{2 \text{ of } 6 + (9 \div 3)\}$ (ii) $5 \text{ of } [8 \div \{6 - (5 - \overline{4-3})\}]$

Solution

(i) $25 - \{2 \text{ of } 6 + (9 \div 3)\} = 25 - \{2 \text{ of } 6 + 3\} = 25 - \{2 \times 6 + 3\}$

$= 25 - \{12 + 3\} = 25 - 15 = 10.$

(ii) Given expression $= 5 \text{ of } [8 \div \{6 - (5 - 1)\}] = 5 \text{ of } [8 \div \{6 - 4\}]$

$= 5 \text{ of } [8 \div 2] = 5 \text{ of } 4 = 5 \times 4 = 20.$

Remember These

1. If two integers are of the same sign then their product and quotient are positive.
   $(+) \times (+) = (+)$, $(-) \times (-) = (+)$, $(+) \div (+) = (+)$, $(-) \div (-) = (+)$.
2. If two integers are of opposite signs then their product and quotient are negative.
   $(+) \times (-) = (-)$, $(-) \times (+) = (-)$, $(+) \div (-) = (-)$, $(-) \div (+) = (-)$.
3. If a be any integer then
   (i) $a \times 1 = a$, (ii) $a \times (-1) = -a$, (iii) $a \times 0 = 0$, (iv) $a \div 1 = a$, (v) $a \div (-1) = -a$, (vi) $0 \div a = 0$.
4. Simplification is carried out in accordance with the rule of BODMAS.

EXERCISE 1D

1. Find the following products.
   (i) $(+12) \times 18$ (ii) $17 \times (+19)$ (iii) $(+37) \times (+44)$

(iv) $(-13) \times (-15)$ (v) $(-28) \times (-40)$ (vi) $(-125) \times (-36)$
(vii) $(-14) \times 37$ (viii) $38 \times (-42)$ (ix) $(-225) \times 40$

2. Complete the following multiplication table.

×	−12	−9	−8	−6	−5	0	5	6	8	9	12
−12											
−9											
−8											
−6											
−5											
0											
5											
6											
8											
9											
12											

Hence, fill in the blank with the sign >, < or =.

(i) $(-12) \times (12)$ ...... $(-12) \times (-12)$ (ii) $(-9) \times (-5)$ ...... $(-9) \times 0$ (iii) $6 \times (-8)$ ...... $9 \times (-5)$

3. Find the following.

(i) $7 \times (-8) \times 12$ (ii) $(-23) \times 37 \times 53$ (iii) $24 \times (-36) \times 48$ (iv) $(-25) \times (+40) \times 0$
(v) $11 \times (-15) \times 17 \times 19$ (vi) $24 \times 30 \times (-28) \times 23$
(vii) $(-2) \times 22 \times (-42) \times (-36) \times 35$ (viii) $7 \times (-9) \times 13 \times (-25) \times 16 \times (+8)$

4. Find the quotient.

(i) $125 \div (+5)$ (ii) $(+36) \div (-4)$ (iii) $625 \div (+25)$
(iv) $(-63) \div (-3)$ (v) $(-128) \div (-16)$ (vi) $(-781) \div (-11)$
(vii) $(-225) \div 9$ (viii) $(-372) \div 12$ (ix) $0 \div (-12)$

5. Simplify the following.

(i) $6 \times 3 - 14 \div 2$ (ii) $2 \times 50 \div (15 - 5) + 8$ (iii) $40 + 12 \div 4 \times 3$
(iv) $30 - \{5 - 36 \div (6 \times 3)\}$ (v) $3 \text{ of } (15 + 9 \div \overline{3 - 2} \times 6)$ (vi) $4 + 4 \div (4 - 4 \div 2)$
(vii) $15 + \{24 - 48 \div (7 + 16 \times 2 - 27)\}$ (viii) $121 \div [47 - \{\overline{15 - 3} \times (7 - 4)\}]$

6. Complete the following using the signs +, −, ×, ÷.

(i) 9 ...... 3 ...... 3 = 9 (ii) 9 ...... 3 ...... 3 = 0
(iii) 9 ...... 3 ...... 3 = 6 (iv) 9 ...... 3 ...... 3 = 24

ANSWERS

1. (i) 216 (ii) 323 (iii) 1628 (iv) 195 (v) 1120 (vi) 4500 (vii) −518 (viii) −1596 (ix) −9000

2. (i) < (ii) > (iii) <

3. (i) −672 (ii) −45103 (iii) −41472 (iv) 0 (v) −53295 (vi) −463680 (vii) −2328480 (viii) 2620800

4. (i) 25 (ii) −9 (iii) 25 (iv) 21 (v) 8 (vi) 71 (vii) −25 (viii) −31 (ix) 0

5. (i) 11 (ii) 18 (iii) 49 (iv) 27 (v) 207 (vi) 6 (vii) 35 (viii) 11

6. (i) +, × or ×, ÷ or +, − or −, + (ii) +, − or −, × (iii) ÷, + (iv) ×, −

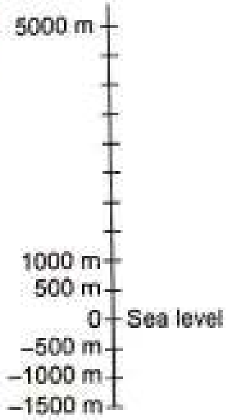
Word Problems

EXAMPLE 1 **At Leh, the temperature was −5 °C on Tuesday and then it dropped by 3 °C on Wednesday. What was the temperature on Wednesday? On Thursday, it rose by 3 °C. What was the temperature on Thursday?**

Solution Temperature on Wednesday = −5 °C − 3 °C = −8 °C.

∴ temperature on Thursday = (temperature on Wednesday) + 3 °C
= −8 °C + 3 °C = −5 °C.

EXAMPLE 2 **A plane is flying at the height of 5250 m above sea level. At a particular point, it is exactly above a submarine floating 1300 m below sea level. What is the vertical distance between them?**



Solution Heights above sea level are considered positive and heights below sea level negative.

∴ 5250 m above sea level means +5250 m = 5250 m,
and 1300 m below sea level means −1300 m.

So, vertical distance between them = 5250 m − (−1300 m)
= 5250 m + 1300 m
= 6550 m.

EXAMPLE 3 **In a quiz, the team A scored 55, −35, −20, 10 and 0, and the team B scored respectively 80, 5, −65, −35 and −10 in five rounds. Which team scored more? By what margin did the winner team win?**

Solution Score of the team A = 55 + (−35) + (−20) + 10 + 0
= 55 − 35 − 20 + 10 + 0 = 10.

Score of the team B = 80 + 5 + (−65) + (−35) + (−10)
= 80 + 5 − 65 − 35 − 10 = −25.

∵ 10 > −25,
∴ the team A scored more.

Difference between their scores = (score of A) − (score of B)
= 10 − (−25) = 10 + 25 = 35.

Hence, the team A won by a margin of 35.

EXAMPLE 4 **A cement company earns a profit of ₹30 per bag of white cement sold and a loss of ₹12 per bag of grey cement sold.**

(i) The company sells 3600 bags of white cement and 6000 bags of grey cement in a month. What is its profit or loss?

(ii) What is the number of white-cement bags it must sell to have neither profit nor loss if the number of grey-cement bags sold is 8000?

Solution (i) Profit for the sale of 3600 bags of white cement
= +₹30 × 3600 = ₹108 000.

Loss for the sale of 6000 bags of grey cement
= −₹12 × 6000 = −₹72 000.

So, profit + loss = ₹108 000 + (−₹72 000)
= ₹108 000 − ₹72 000 = ₹36 000, which is positive.

∴ there is a profit.

So, the company makes a profit of ₹36 000.

(ii) Loss for the sale of 8000 bags of grey cement

$= -₹12 \times 8000 = -₹96\,000.$

Let the company sell x bags of white cement so that there is no loss.

$\therefore$ profit for the sale of x bags of white cement $= +₹30 \times x = ₹30x$.

$\because$ profit + sale = 0,

$\therefore$ $₹30x + (-₹96\,000) = 0$

or $₹30x - ₹96\,000 = 0$

or $₹30x = ₹96\,000$.

$\therefore$ $x = \dfrac{₹96\,000}{₹30} = 3200.$

Hence, the required number of bags of white cement to be sold is 3200.

EXAMPLE 5 **In a class test containing 12 questions, 4 marks are given for every correct answer and −2 marks for every incorrect answer. (i) Sumit attempts all questions, but only 8 are correct. What is his total score? (ii) Rajni attempts 10 questions, but only 5 of her answers are correct. What is her total score?**

Solution (i) Marks for 8 correct answers $= 8 \times 4 = 32$.

Marks for 4 incorrect answers $= 4 \times (-2) = -8$.

$\therefore$ total score of Sumit $= 32 + (-8) = 32 - 8 = 24$.

(ii) Marks for 5 correct answers $= 5 \times 4 = 20$.

Marks for 5 incorrect answers $= 5 \times (-2) = -10$.

$\therefore$ total score of Rajni $= 20 + (-10) = 20 - 10 = 10$.

EXERCISE 1E

1. At Srinagar, the temperature was −4 °C on Monday and then it dropped by 2 °C on Tuesday. What was the temperature of Srinagar on Tuesday? On Wednesday it rose by 5 °C. What was the temperature on this day?
2. Ramesh deposits ₹5000 in his bank account and next day withdraws ₹3780 from it. Find the balance in his account after the withdrawal.
3. A plane is flying at the height of 6300 m above sea level. At a particular point, it is exactly above a submarine floating 1400 m below sea level. Find the vertical distance between them.
4. A submarine is floating 50 m below sea level.
   (i) It goes down a further 380 m to a point A. Write down the depth of the point A below sea level.
   (ii) From the point A, it rises 120 m to another point B. What is the depth of B below sea level?
5. In a hotel, food is stored in a freezer at −20 °C.
   (i) What would be the temperature if it was raised by 5 °C?
   (ii) What would be the temperature if it was lowered by 0.5 °C every hour for 12 hours?
6. In a quiz, the team X scored −40, 90, 0, 30 and −70, and the team Y scored respectively 75, 30, −50, −70 and 10 in five rounds.

(i) Find the scores of the two teams.

(ii) What was the margin by which the victorious team won?

7. On a certain day at a certain time in Siberia, the temperature was −28 °C. On the same day at the same time, the temperature in Brazil was 28 °C. What is the difference between these two temperatures?

8. A certain freezing process requires that the room temperature be lowered from 45 °C at the rate of 4 °C every hour. What will be the room temperature 10 hours after the process begins?

9. In a class test containing 15 questions, 5 marks are awarded for every correct answer, −2 marks for every incorrect answer, and 0 marks for questions not attempted.

(i) Raju gets 6 correct and 6 incorrect answers. What is his score?

(ii) Manish gets 7 correct and 8 incorrect answers. What is his score?

10. A shopkeeper gains ₹2 on each pen and loses 50 paise on each pencil. He sells 60 pens and 100 pencils. How much has he gained or lost?

11. An elevator descends into a mine shaft at the rate of 6 m/min. The descent starts from 10 m above sea level.

(i) Where will it reach in 20 minutes?

(ii) How long will it take to reach a depth of −350 m?

ANSWERS

1. −6 °C, −1 °C

2. ₹1220

3. 7700 m

4. (i) 430 m (ii) 310 m

5. (i) −15 °C (ii) −26 °C

6. (i) (Score of X) = 10 and (score of Y) = −5 (ii) 15

7. 56 °C

8. 5 °C

9. (i) 18 (ii) 19

10. Gain of ₹70

11. (i) −110 m (i.e., 110 m below sea level) (ii) 1 hour

❖

Fractions represent parts of a whole. For example, if an apple is divided into three equal parts, each part is called one third and is denoted by $\frac{1}{3}$. We call $\frac{1}{3}$ a fraction. Similarly, $\frac{1}{4}, \frac{2}{5}, \frac{3}{7}$ and $\frac{5}{4}$ are also fractions.

In any fraction $\frac{a}{b}$, the dividend a is called the numerator and the divisor b is called the denominator. For example, in the fraction $\frac{2}{7}$, the numerator is 2 and the denominator is 7.

Classification of Fractions

Common (or simple or vulgar) fraction

The numerator and the denominator are integers in a common fraction, also known as a simple fraction or vulgar fraction.

Examples $\frac{3}{5}, \frac{7}{9}, \frac{11}{7}$ and $\frac{3}{2}$ are some simple fractions.

Complex fraction

A fraction in which the numerator or the denominator or both of them also contain fractions is called a complex fraction.

Examples $\frac{2/3}{5}, \frac{3}{5/7}$ and $\frac{2/3}{8/9}$ are some complex fractions.

Proper fraction

A vulgar fraction in which the numerator is less than the denominator is called a proper fraction.

Examples $\frac{1}{2}, \frac{2}{3}, \frac{5}{6}$ and $\frac{8}{9}$ are some proper fractions.

Improper fraction

A vulgar fraction in which the numerator is greater than the denominator is called an improper fraction.

Examples $\frac{3}{2}, \frac{8}{5}, \frac{9}{8}$ and $\frac{120}{11}$ are some improper fractions.

Mixed fraction

An integer together with a proper fraction is called a mixed fraction.

Examples $1\frac{1}{2}, 8\frac{9}{11}$ and $15\frac{14}{29}$ are some mixed fractions.

A mixed fraction can easily be converted into an improper fraction as follows.

$$\text{Mixed fraction} = \frac{\text{(integral part)} \times \text{(denominator of the proper fraction)} + \text{(numerator of the proper fraction)}}{\text{denominator}}.$$

Examples (i) $8\frac{9}{11} = \frac{8 \times 11 + 9}{11} = \frac{97}{11}$. (ii) $15\frac{14}{29} = \frac{15 \times 29 + 14}{29} = \frac{449}{29}$.

An improper fraction can be converted into a mixed fraction as follows.

$$\text{Mixed fraction} = \text{quotient in (numerator} \div \text{denominator)} + \frac{\text{remainder}}{\text{denominator}}.$$

Examples (i) $\frac{11}{7} = 1 + \frac{4}{7} = 1\frac{4}{7}$ because in $11 \div 7$, quotient $= 1$ and remainder $= 4$.

(ii) $\frac{43}{6} = 7 + \frac{1}{6} = 7\frac{1}{6}$ because in $43 \div 6$, quotient $= 7$ and remainder $= 1$.

Equivalent (or equal) fractions

If the numerator and the denominator of a fraction are multiplied or divided by the same nonzero number then we get an equivalent, or equal, fraction.

Examples (i) $\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}$, $\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$, $\frac{3}{4} = \frac{3 \times 4}{4 \times 4} = \frac{12}{16}$, etc.

(ii) $\frac{20}{24} = \frac{20 \div 2}{24 \div 2} = \frac{10}{12}$, $\frac{20}{24} = \frac{20 \div 4}{24 \div 4} = \frac{5}{6}$.

This brings us to the conclusion that the value of a fraction remains the same if both its numerator and denominator are multiplied or divided by the same number.

Consider the set of equivalent fractions $\frac{3}{4}$ and $\frac{6}{8}$, and also the set of $\frac{10}{12}$ and $\frac{5}{6}$.

Clearly, $3 \times 8 = 4 \times 6 = 24$ and $10 \times 6 = 5 \times 12 = 60$.

We can generalise this as follows.

Two fractions $\frac{a}{b}$ and $\frac{c}{d}$ are equivalent fractions if $ad = bc$.

EXAMPLE **Verify whether $\frac{4}{5}$ and $\frac{16}{20}$ are equivalent fractions.**

Solution $4 \times 20 = 80$ and $5 \times 16 = 80$.

So, $\frac{4}{5}$ and $\frac{16}{20}$ are equivalent fractions.

EXAMPLE **Fill in the blanks.** **(i) $\frac{2}{7} = \frac{\cdots\cdots}{56}$** **(ii) $\frac{3}{11} = \frac{15}{\cdots\cdots}$**

Solution (i) $\frac{2}{7} = \frac{\text{required number}}{56}$.

$\therefore$ (required number) $\times 7 = 2 \times 56$.

$\therefore$ required number $= \frac{2 \times \cancel{56}^{8}}{\cancel{7}_{1}} = 16$.

(ii) $\frac{3}{11} = \frac{15}{\text{required number}}$.

$\therefore$ $3 \times$ (required number) $= 15 \times 11$.

$\therefore$ required number $= \frac{\cancel{15}^{5} \times 11}{\cancel{3}_{1}} = 55$.

Simplest form of a fraction

A fraction is said to be in the simplest form if its numerator and denominator have no factor in common except 1.

Examples (i) $\frac{2}{3}, \frac{3}{7}, \frac{7}{8}, \frac{101}{103}$ and $\frac{9}{7}$ are some fractions in the simplest form.

(ii) $\frac{3}{6}, \frac{18}{36}$ and $\frac{100}{15}$ are some fractions which are not in the simplest form.

A fraction can be converted into its simplest form by cancelling out the common factors in the numerator and the denominator. This can be done in two ways: (i) by prime factorisation and (ii) by HCF.

EXAMPLE **Write the fraction $\frac{270}{315}$ in the simplest form.**

Solution *By prime factorisation*

$270 = 2 \times 135 = 2 \times 3 \times 45 = 2 \times 3 \times 3 \times 15 = 2 \times 3 \times 3 \times 3 \times 5$

and $315 = 3 \times 105 = 3 \times 3 \times 35 = 3 \times 3 \times 5 \times 7$.

$\therefore \quad \frac{270}{315} = \frac{2 \times \cancel{3}^{1} \times \cancel{3}^{1} \times 3 \times \cancel{5}^{1}}{\cancel{3}_{1} \times \cancel{3}_{1} \times \cancel{5}_{1} \times 7} = \frac{2 \times 3}{7} = \frac{6}{7}$.

By HCF

The HCF of 270 and 315 is 45.

$\therefore \quad \frac{270}{315} = \frac{270 \div 45}{315 \div 45} = \frac{6}{7}$.

```
270) 315 (1
   - 270
     45) 270 (6
       - 270
           ×
```

Like and unlike fractions

Two or more fractions are called like fractions if they have the same denominator. Fractions that do not have the same denominator are called unlike fractions.

Examples (i) $\frac{2}{7}, \frac{5}{7}$ and $\frac{6}{7}$ are like fractions.

(ii) $\frac{2}{7}, \frac{3}{5}$ and $\frac{4}{9}$ are unlike fractions.

Conversion of unlike fractions into like fractions

Take the following steps to convert unlike fractions into like fractions.

Steps: 1. Find the LCM of the denominators of the fractions.

2. Divide the LCM by the denominators of the fractions.

3. Multiply the numerator as well as the denominator of each fraction by the corresponding quotient.

EXAMPLE **Convert $\frac{5}{12}$, $\frac{7}{16}$ and $\frac{11}{18}$ into like fractions.**

Solution

2	12, 16, 18
2	6, 8, 9
3	3, 4, 9
	1, 4, 3

$\therefore$ LCM of the denominators $= 2 \times 2 \times 3 \times 4 \times 3 = 144$.

Now, $144 \div 12 = 12$, $144 \div 16 = 9$, $144 \div 18 = 8$.

$\therefore \quad \frac{5}{12} = \frac{5 \times 12}{12 \times 12} = \frac{60}{144}, \quad \frac{7}{16} = \frac{7 \times 9}{16 \times 9} = \frac{63}{144}, \quad \frac{11}{18} = \frac{11 \times 8}{18 \times 8} = \frac{88}{144}$.

So, the like fractions are $\frac{60}{144}$, $\frac{63}{144}$ and $\frac{88}{144}$ respectively.

Comparison of fractions

Two or more fractions can be compared if they are like fractions. If the given fractions are unlike fractions then convert them into like fractions. In the case of two like fractions, the one with the bigger numerator is greater.

EXAMPLE **Compare the fractions $\frac{7}{17}$, $\frac{15}{17}$ and $\frac{13}{17}$.**

Solution The given fractions are like fractions.

$\because \quad 15 > 13 > 7$,

$\therefore \quad \frac{15}{17} > \frac{13}{17} > \frac{7}{17}$.

EXAMPLE **Compare the fractions $\frac{4}{15}$, $\frac{3}{5}$ and $\frac{7}{30}$.**

Solution To change the given fractions into like fractions, we have to find the LCM of 15, 5 and 30.

3	15, 5, 30
5	5, 5, 10
	1, 1, 2

$\therefore \quad$ LCM $= 3 \times 5 \times 2 = 30$.

Now, $30 \div 15 = 2$, $30 \div 5 = 6$, $30 \div 30 = 1$.

$\therefore \quad \frac{4}{15} = \frac{4 \times 2}{15 \times 2} = \frac{8}{30}, \quad \frac{3}{5} = \frac{3 \times 6}{5 \times 6} = \frac{18}{30}, \quad \frac{7}{30} = \frac{7}{30}$.

$\because \quad 18 > 8 > 7$,

$\therefore \quad \frac{18}{30} > \frac{8}{30} > \frac{7}{30}$ or $\frac{3}{5} > \frac{4}{15} > \frac{7}{30}$.

Solved Examples

EXAMPLE 1 **Reduce each of the following to its simplest form.**

(i) $\frac{476}{532}$ (ii) $\frac{546}{714}$

Solution (i) $476 = 2 \times 238 = 2 \times 2 \times 119 = 2 \times 2 \times 7 \times 17$

and $532 = 2 \times 266 = 2 \times 2 \times 133 = 2 \times 2 \times 7 \times 19$.

$\therefore \frac{476}{532} = \frac{2 \times 2 \times 7 \times 17}{2 \times 2 \times 7 \times 19} = \frac{17}{19}$.

(ii) The HCF of 546 and 714 is 42.

$\therefore \frac{546}{714} = \frac{546 \div 42}{714 \div 42} = \frac{13}{17}$.

546) 714 (1
 − 546
 168) 546 (3
 − 504
 42) 168 (4
 − 168
 ×

EXAMPLE 2 **Convert the following fractions into like fractions.**

$\frac{11}{24}, \frac{5}{12}, \frac{7}{18}, \frac{13}{45}$

Solution

2	24, 12, 18, 45
2	12, 6, 9, 45
3	6, 3, 9, 45
3	2, 1, 3, 15
	2, 1, 1, 5

$\therefore$ LCM of the denominators $= 2 \times 2 \times 3 \times 3 \times 2 \times 5 = 360$.

Now, $360 \div 24 = 15$, $360 \div 12 = 30$, $360 \div 18 = 20$ and $360 \div 45 = 8$.

$\therefore \frac{11}{24} = \frac{11 \times 15}{24 \times 15} = \frac{165}{360}$, $\frac{5}{12} = \frac{5 \times 30}{12 \times 30} = \frac{150}{360}$,

$\frac{7}{18} = \frac{7 \times 20}{18 \times 20} = \frac{140}{360}$ and $\frac{13}{45} = \frac{13 \times 8}{45 \times 8} = \frac{104}{360}$.

Hence, the required like fractions are $\frac{165}{360}, \frac{150}{360}, \frac{140}{360}$ and $\frac{104}{360}$.

EXAMPLE 3 **Write the following fractions in the ascending order.**

$\frac{5}{6}, \frac{7}{8}, \frac{11}{12}, \frac{13}{15}, \frac{23}{36}$

Solution To change the fractions into like fractions, we must find the LCM of the denominators.

2	6, 8, 12, 15, 36
2	3, 4, 6, 15, 18
3	3, 2, 3, 15, 9
	1, 2, 1, 5, 3

$\therefore$ LCM of the denominators $= 2 \times 2 \times 3 \times 2 \times 5 \times 3 = 360$.

Now, $360 \div 6 = 60$, $360 \div 8 = 45$, $360 \div 12 = 30$, $360 \div 15 = 24$ and $360 \div 36 = 10$.

$\therefore \quad \frac{5}{6} = \frac{5 \times 60}{6 \times 60} = \frac{300}{360}, \quad \frac{7}{8} = \frac{7 \times 45}{8 \times 45} = \frac{315}{360}, \quad \frac{11}{12} = \frac{11 \times 30}{12 \times 30} = \frac{330}{360},$

$\frac{13}{15} = \frac{13 \times 24}{15 \times 24} = \frac{312}{360}$ and $\frac{23}{36} = \frac{23 \times 10}{36 \times 10} = \frac{230}{360}.$

$\because \quad 230 < 300 < 312 < 315 < 330.$

$\therefore \quad \frac{230}{360} < \frac{300}{360} < \frac{312}{360} < \frac{315}{360} < \frac{330}{360}$ or $\frac{23}{36} < \frac{5}{6} < \frac{13}{15} < \frac{7}{8} < \frac{11}{12}.$

Remember These

1. To reduce a fraction to its simplest form, cancel out the common factors in the numerator and the denominator.
2. To convert two or more fractions into like fractions, multiply the numerator and the denominator of each fraction by the corresponding quotient in {(LCM of the denominators of all fractions) ÷ (denominator of the fraction)}.
3. To compare two fractions, change them into like fractions. After this, the fraction with the larger numerator will be bigger.

EXERCISE 2A

1. Express the following as mixed fractions.

(i) $\frac{37}{12}$ (ii) $\frac{115}{18}$ (iii) $\frac{316}{23}$ (iv) $\frac{947}{31}$

2. Express the following as improper fractions.

(i) $3\frac{7}{12}$ (ii) $9\frac{11}{43}$ (iii) $7\frac{26}{37}$ (iv) $18\frac{101}{107}$

3. Fill in the blanks.

(i) $\frac{4}{9} = \frac{\ldots\ldots}{36}$ (ii) $\frac{5}{11} = \frac{40}{\ldots\ldots}$ (iii) $\frac{\ldots\ldots}{55} = 5\frac{7}{11}$ (iv) $\frac{219}{\ldots\ldots} = 2\frac{3}{35}$

4. Write each of the following fractions in the simplest form.

(i) $\frac{144}{336}$ (ii) $\frac{150}{525}$ (iii) $\frac{276}{322}$ (iv) $\frac{203}{259}$

5. Write the fractions in the descending order in each of the following.

(i) $\frac{1}{2}, \frac{7}{8}, \frac{11}{12}$ (ii) $\frac{7}{20}, \frac{8}{15}, \frac{3}{10}$ (iii) $\frac{8}{11}, \frac{29}{44}, \frac{37}{132}$ (iv) $\frac{2}{7}, \frac{3}{5}, \frac{9}{10}, \frac{13}{20}, \frac{23}{35}$

6. Arrange the fractions in the ascending order in each of the following.

(i) $\frac{7}{16}, \frac{5}{12}, \frac{17}{32}, \frac{23}{36}$ (ii) $\frac{17}{48}, \frac{19}{36}, \frac{11}{30}, \frac{5}{24}$ (iii) $\frac{4}{5}, \frac{12}{35}, \frac{17}{21}, \frac{13}{30}, \frac{101}{105}$

ANSWERS

1. (i) $3\frac{1}{12}$ (ii) $6\frac{7}{18}$ (iii) $13\frac{17}{23}$ (iv) $30\frac{17}{31}$

2. (i) $\frac{43}{12}$ (ii) $\frac{398}{43}$ (iii) $\frac{285}{37}$ (iv) $\frac{2027}{107}$

3. (i) 16 (ii) 88 (iii) 310 (iv) 105

4. (i) $\frac{3}{7}$ (ii) $\frac{2}{7}$ (iii) $\frac{6}{7}$ (iv) $\frac{29}{37}$

5. (i) $\frac{11}{12}, \frac{7}{8}, \frac{1}{2}$ (ii) $\frac{8}{15}, \frac{7}{20}, \frac{3}{10}$ (iii) $\frac{8}{11}, \frac{29}{44}, \frac{37}{132}$ (iv) $\frac{9}{10}, \frac{23}{35}, \frac{13}{20}, \frac{3}{5}, \frac{2}{7}$

6. (i) $\frac{5}{12}, \frac{7}{16}, \frac{17}{32}, \frac{23}{36}$ (ii) $\frac{5}{24}, \frac{17}{48}, \frac{11}{30}, \frac{19}{36}$ (iii) $\frac{12}{35}, \frac{13}{30}, \frac{4}{5}, \frac{17}{21}, \frac{101}{105}$

Fundamental Operations with Fractions

Addition/subtraction of fractions

Take the following steps to add or subtract two or more fractions.

Steps
1. Convert the mixed fractions (if any) into improper fractions.
2. Convert all the fractions into like fractions.
3. Add the numerators of the like fractions obtained in Step 2.
4. Write a fraction with the number obtained in Step 3 as the numerator and the common denominator in Step 2 as the denominator.
5. Reduce the fraction obtained in Step 4 to its simplest form. If it is an improper fraction, express it as a mixed fraction.

Short-cut formula

Sum of two fractions = $\dfrac{\text{(numerator of the first)} \times \text{(LCM of the denominators)} \div \text{(denominator of the first)} + \text{(numerator of the second)} \times \text{(LCM of the denominators)} \div \text{(denominator of the second)}}{\text{LCM of the denominators}}$.

Notes
- This rule can be extended to three or more fractions.
- The rule is similar for subtraction. Replace '+' with '–'.

EXAMPLE **Find $\frac{3}{4} + \frac{7}{8} + \frac{11}{15}$.**

Solution The LCM of 4, 8 and 15 is $2 \times 2 \times 2 \times 15 = 120$.

2	4, 8, 15
2	2, 4, 15
	1, 2, 15

Now, $120 \div 4 = 30$, $120 \div 8 = 15$ and $120 \div 15 = 8$.

$\therefore \quad \frac{3}{4} = \frac{3 \times 30}{4 \times 30} = \frac{90}{120}$, $\frac{7}{8} = \frac{7 \times 15}{8 \times 15} = \frac{105}{120}$ and $\frac{11}{15} = \frac{11 \times 8}{15 \times 8} = \frac{88}{120}$.

$\therefore \quad \frac{3}{4} + \frac{7}{8} + \frac{11}{15} = \frac{90}{120} + \frac{105}{120} + \frac{88}{120} = \frac{90 + 105 + 88}{120} = \frac{283}{120} = 2\frac{43}{120}$.

Short cut

The LCM of 4, 8 and 15 is 120.

$$\therefore \quad \frac{3}{4}+\frac{7}{8}+\frac{11}{15}=\frac{3\times(120\div4)+7\times(120\div8)+11\times(120\div15)}{120}$$

$$=\frac{3\times30+7\times15+11\times8}{120}=\frac{90+105+88}{120}=\frac{283}{120}=2\frac{43}{120}.$$

EXAMPLE **Find** $\frac{3}{7}-\frac{1}{14}-\frac{5}{21}$.

Solution The LCM of 7, 14 and 21 is $7\times2\times3=42$.

7	7, 14, 21
	1, 2, 3

$$\therefore \quad \frac{3}{7}-\frac{1}{14}-\frac{5}{21}=\frac{3\times(42\div7)-1\times(42\div14)-5\times(42\div21)}{42}$$

$$=\frac{3\times6-1\times3-5\times2}{42}=\frac{18-3-10}{42}=\frac{5}{42}.$$

EXAMPLE **Find** $\frac{3}{8}-\frac{1}{2}+\frac{7}{36}$.

Solution The LCM of 8, 2 and 36 is $2\times2\times2\times9=72$.

Now, $72\div8=9$, $72\div2=36$, $72\div36=2$.

2	8, 2, 36
2	4, 1, 18
	2, 1, 9

$$\therefore \quad \frac{3}{8}=\frac{3\times9}{8\times9}=\frac{27}{72},\quad \frac{1}{2}=\frac{1\times36}{2\times36}=\frac{36}{72},\quad \frac{7}{36}=\frac{7\times2}{36\times2}=\frac{14}{72}.$$

$$\therefore \quad \frac{3}{8}-\frac{1}{2}+\frac{7}{36}=\frac{27}{72}-\frac{36}{72}+\frac{14}{72}=\frac{27-36+14}{72}=\frac{5}{72}.$$

Short cut

The LCM of 8, 2 and 36 is 72.

$$\therefore \quad \frac{3}{8}-\frac{1}{2}+\frac{7}{36}=\frac{3\times(72\div8)-1\times(72\div2)+7\times(72\div36)}{72}$$

$$=\frac{3\times9-1\times36+7\times2}{72}=\frac{27-36+14}{72}=\frac{5}{72}.$$

Multiplication of fractions

To multiply two or more fractions, take the following steps.

Steps
1. Convert the mixed fractions (if any) into improper fractions.
2. Multiply the numerators of all the fractions to form the numerator of the product.
3. Multiply the denominators of all the fractions to form the denominator of the product.
4. Reduce the product to its simplest form by cancelling the common factors from the numerator and denominator.

$$\text{Product of fractions}=\frac{\text{product of the numerators of all the fractions}}{\text{product of the denominators of all the fractions}}.$$

Examples (i) $\frac{5}{6}\times\frac{3}{8}=\frac{5\times3^{1}}{6_{2}\times8}=\frac{5\times1}{2\times8}=\frac{5}{16}.$

(ii) $6\frac{3}{8} \times 2\frac{12}{17} = \frac{51^{3}}{8_{4}} \times \frac{46^{23}}{17_{1}} = \frac{3 \times 23}{4 \times 1} = \frac{69}{4} = 17\frac{1}{4}$.

(iii) $\frac{5}{7} \times \frac{3}{4} \times 2\frac{4}{5} = \frac{5^{1}}{7_{1}} \times \frac{3}{4_{2}} \times \frac{14^{2^{1}}}{5_{1}} = \frac{1 \times 3 \times 1}{1 \times 2 \times 1} = \frac{3}{2} = 1\frac{1}{2}$.

Fraction as the operator 'of'

The operator 'of ' effectively means 'multiplied by'.

Suppose Anshaj gives $\frac{1}{3}$ of his 6 chocolates to Aditya.

∴ Aditya has $\left(\frac{1}{3} \text{ of } 6\right)$ chocolates, i.e., $\frac{1}{3} \times 6 = 2$ chocolates.

Similarly, $\frac{1}{2}$ of $18 = \frac{1}{2} \times 18 = 9$ and $\frac{3}{4}$ of $72 = \frac{3}{4} \times 72 = 54$.

So, $\frac{p}{q}$ of $x = \frac{p}{q} \times x$.

Examples (i) $\frac{7}{8}$ of $40 = \frac{7}{8_{1}} \times 40^{5} = \frac{7 \times 5}{1} = 35$.

(ii) $\frac{9}{11}$ of $220 = \frac{9}{11_{1}} \times 220^{20} = \frac{9 \times 20}{1} = 180$.

(iii) $\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

(iv) $\frac{2}{3}$ of $\frac{1}{2} = \frac{2^{1}}{3} \times \frac{1}{2_{1}} = \frac{1}{3}$.

EXAMPLE **Find the following.**

(i) $\frac{1}{2}$ of a rupee **(ii) $\frac{3}{4}$ of a year** **(iii) $\frac{2}{3}$ of a day**

(iv) $\frac{2}{5}$ of one litre **(v) $\frac{1}{4}$ of 2 kg** **(vi) $\frac{1}{4}$ of an hour**

Solution (i) $\frac{1}{2}$ of a rupee $= \frac{1}{2}$ of 100 paise $= \frac{1}{2} \times 100$ paise $=$ 50 paise.

(ii) $\frac{3}{4}$ of a year $= \frac{3}{4}$ of 12 months $= \frac{3}{4} \times 12$ months $=$ 9 months.

(iii) $\frac{2}{3}$ of a day $= \frac{2}{3}$ of 24 hours $= \frac{2}{3} \times 24$ hours $=$ 16 hours.

(iv) $\frac{2}{5}$ of one litre $= \frac{2}{5}$ of 1000 mL $= \frac{2}{5} \times 1000$ mL $=$ 400 mL.

(v) $\frac{1}{4}$ of 2 kg $= \frac{1}{4}$ of 2000 g $= \frac{1}{4} \times 2000$ g $=$ 500 g.

(vi) $\frac{1}{4}$ of an hour $= \frac{1}{4}$ of 60 min $= \frac{1}{4} \times 60$ min $=$ 15 min.

Reciprocal of a number

A number is called the reciprocal of another number if the product of the two numbers is 1.

Examples (i) $9 \times \frac{1}{9} = 1$. ∴ 9 is the reciprocal of $\frac{1}{9}$, and $\frac{1}{9}$ is the reciprocal of 9.

(ii) $\frac{2}{7} \times \frac{7}{2} = 1$. ∴ $\frac{2}{7}$ is the reciprocal of $\frac{7}{2}$, and $\frac{7}{2}$ is the reciprocal of $\frac{2}{7}$.

Method To find the reciprocal of a number, write the number as a proper or an improper fraction and then interchange the numerator and denominator, retaining the minus (–) sign, if any, in the numerator.

Examples (i) Reciprocal of $5 \left(\text{i.e.}, \frac{5}{1}\right)$ is $\frac{1}{5}$. (ii) Reciprocal of $7\frac{4}{5} \left(\text{i.e.}, \frac{39}{5}\right)$ is $\frac{5}{39}$.

(iii) Reciprocal of $-3\frac{8}{11} \left(\text{i.e.}, -\frac{41}{11}\right)$ is $-\frac{11}{41}$.

Division of fractions

To divide a fraction by another fraction, take the following steps.

Steps 1. Convert the mixed fractions (if any) into improper fractions.

2. Multiply the dividend and the reciprocal of the divisor.

3. Reduce the product to its simplest form.

Examples (i) $\frac{2}{3} \div \frac{4}{7} = \frac{2^1}{3} \times \frac{7}{4_2} = \frac{1 \times 7}{3 \times 2} = \frac{7}{6} = 1\frac{1}{6}$.

(ii) $8\frac{9}{17} \div 14\frac{1}{2} = \frac{145}{17} \div \frac{29}{2} = \frac{145^5}{17} \times \frac{2}{29_1} = \frac{5 \times 2}{17 \times 1} = \frac{10}{17}$.

Simplification of complex fractions

Simplifying a complex fraction is like carrying out a division.

Examples (i) $\dfrac{3\frac{1}{2}}{4\frac{1}{5}} = \dfrac{\frac{7}{2}}{\frac{21}{5}} = \frac{7}{2} \div \frac{21}{5} = \frac{7^1}{2} \times \frac{5}{21_3} = \frac{1 \times 5}{2 \times 3} = \frac{5}{6}$.

(ii) $\dfrac{\frac{2}{3}}{5} = \frac{2}{3} \div 5 = \frac{2}{3} \times \frac{1}{5} = \frac{2}{3 \times 5} = \frac{2}{15}$.

(iii) $\dfrac{6}{\frac{8}{9}} = 6 \div \frac{8}{9} = 6^3 \times \frac{9}{8_4} = \frac{3 \times 9}{4} = \frac{27}{4} = 6\frac{3}{4}$.

Simplification of an expression

An expression involving fractions is simplified by following the rule of BODMAS. Brackets are removed in this order: line bracket, first brackets, second brackets, third brackets.

EXAMPLE **Simplify** $8\frac{1}{4}$ **of** $\frac{3}{4} + 6 \div \left[4 \times \left\{3 - \left(2 - \frac{1}{3}\right)\right\}\right]$.

Solution Given expression $= \frac{33}{4} \text{ of } \frac{3}{4} + 6 \div \left[4 \times \left\{3 - \frac{6-1}{3}\right\}\right]$

$$= \frac{33}{4} \text{ of } \frac{3}{4} + 6 \div \left[4 \times \left\{3 - \frac{5}{3}\right\}\right] = \frac{33}{4} \text{ of } \frac{3}{4} + 6 \div \left[4 \times \left\{\frac{3 \times 3 - 5}{3}\right\}\right]$$

$$= \frac{33}{4} \text{ of } \frac{3}{4} + 6 \div \left[4 \times \frac{4}{3}\right] = \frac{33}{4} \text{ of } \frac{3}{4} + 6 \div \frac{16}{3}$$

$$= \frac{33}{4} \times \frac{3}{4} + 6 \times \frac{3}{16} = \frac{99}{16} + \frac{18}{16} = \frac{99+18}{16} = \frac{117}{16} = 7\frac{5}{16}.$$

Solved Examples

EXAMPLE 1 **Simplify** $6\frac{2}{3} - 3\frac{3}{4} + 4\frac{5}{6} - \frac{7}{15}$.

Solution $6\frac{2}{3} - 3\frac{3}{4} + 4\frac{5}{6} - \frac{7}{15} = \frac{20}{3} - \frac{15}{4} + \frac{29}{6} - \frac{7}{15}$

$$= \frac{20 \times (60 \div 3) - 15 \times (60 \div 4) + 29 \times (60 \div 6) - 7 \times (60 \div 15)}{60}$$

$$= \frac{20 \times 20 - 15 \times 15 + 29 \times 10 - 7 \times 4}{60}$$

$$= \frac{400 - 225 + 290 - 28}{60} = \frac{437}{60} = 7\frac{17}{60}.$$

2	3, 4, 6, 15
3	3, 2, 3, 15
	1, 2, 1, 5

EXAMPLE 2 **Find** $11\frac{7}{8}$ **of** $5\frac{31}{45} \div 76$.

Solution $11\frac{7}{8} \text{ of } 5\frac{31}{45} \div 76 = \frac{95}{8} \text{ of } \frac{256}{45} \div 76 = \frac{95^{5}}{8_{1}} \times \frac{256^{32^{8}}}{45_{9}} \times \frac{1}{76_{4_{1}}} = \frac{8}{9}.$

EXAMPLE 3 **Simplify** $\left(3\frac{1}{4} \times 4\frac{1}{3}\right) \div \left(1\frac{4}{5} + 1\frac{2}{3}\right) \times \left(2\frac{1}{13} - 1\frac{1}{2}\right)$.

Solution Given expression $= \left(\frac{13}{4} \times \frac{13}{3}\right) \div \left(\frac{9}{5} + \frac{5}{3}\right) \times \left(\frac{27}{13} - \frac{3}{2}\right)$

$$= \left(\frac{13}{4} \times \frac{13}{3}\right) \div \left(\frac{9 \times 3 + 5 \times 5}{5 \times 3}\right) \times \left(\frac{27 \times 2 - 3 \times 13}{13 \times 2}\right)$$

$$= \frac{13 \times 13}{4 \times 3} \div \frac{52}{15} \times \frac{15}{13 \times 2}$$

$$= \frac{13^{1} \times 13^{1}}{4 \times 3_{1}} \times \frac{15^{5}}{52_{4}} \times \frac{15}{13_{1} \times 2} = \frac{5 \times 15}{4 \times 4 \times 2} = \frac{75}{32} = 2\frac{11}{32}.$$

EXAMPLE 4 **Simplify** $\left[2\frac{1}{12} \times \left\{12\frac{1}{5} \div \left(7\frac{1}{4} - 2\frac{1}{6}\right)\right\}\right] \div 6\frac{2}{3}$.

Solution Given expression $= \left[\frac{25}{12} \times \left\{\frac{61}{5} \div \left(\frac{29}{4} - \frac{13}{6}\right)\right\}\right] \div \frac{20}{3}$

$= \left[\frac{25}{12} \times \left\{\frac{61}{5} \div \frac{29 \times 3 - 13 \times 2}{12}\right\}\right] \div \frac{20}{3}$

$= \left[\frac{25}{12} \times \left\{\frac{61}{5} \div \frac{87 - 26}{12}\right\}\right] \div \frac{20}{3} = \left[\frac{25}{12} \times \left\{\frac{61^1}{5} \times \frac{12}{61_1}\right\}\right] \div \frac{20}{3}$

$= \left[\frac{25^5}{12_1} \times \frac{12^1}{5_1}\right] \div \frac{20}{3} = 5^1 \times \frac{3}{20_4} = \frac{3}{4}$.

EXAMPLE 5 **Simplify** $6\frac{3}{7}$ **of** $\left[3 - \left\{\frac{3}{5} + \left(\frac{5}{18} \div \frac{2}{3} - \frac{1}{2}\right)\right\}\right]$.

Solution Given expression $= \frac{45}{7}$ of $\left[3 - \left\{\frac{3}{5} + \left(\frac{5}{18} \div \frac{2 \times 2 - 3}{6}\right)\right\}\right]$

$= \frac{45}{7}$ of $\left[3 - \left\{\frac{3}{5} + \left(\frac{5}{18} \div \frac{1}{6}\right)\right\}\right] = \frac{45}{7}$ of $\left[3 - \left\{\frac{3}{5} + \frac{5}{18_3} \times 6^1\right\}\right]$

$= \frac{45}{7}$ of $\left[3 - \left\{\frac{3}{5} + \frac{5}{3}\right\}\right] = \frac{45}{7}$ of $\left[3 - \frac{9 + 25}{5 \times 3}\right] = \frac{45}{7}$ of $\left[3 - \frac{34}{15}\right]$

$= \frac{45}{7}$ of $\frac{45 - 34}{15} = \frac{45^3}{7} \times \frac{11}{15_1} = \frac{33}{7} = 4\frac{5}{7}$.

Remember These

1. To add two fractions, change them into like fractions and then add. Alternatively, use the following formula.

$$\text{Sum of two fractions} = \frac{\begin{array}{c}\text{(numerator of the first)} \times \text{(LCM of the denominators)} \\ \div \text{(denominator of the first)} + \text{(numerator of the second)} \\ \times \text{(LCM of the denominators)} \div \text{(denominator of the second)}\end{array}}{\text{LCM of the denominators}}.$$

2. To subtract one fraction from another, replace '+' by '−' in the formula for addition.
3. Product of two or more fractions $= \frac{\text{product of the numerators of all fractions}}{\text{product of the denominators of all fractions}}$.
4. To divide a fraction by another fraction, multiply the dividend by the reciprocal of the divisor.
5. To simplify fractions, use the rule of BODMAS.

EXERCISE 2B

1. Simplify the following.

(i) $4\frac{9}{20} + \frac{7}{12} - 1\frac{13}{15}$ (ii) $\frac{3}{4} + \frac{1}{2} - \frac{5}{8}$ (iii) $2\frac{1}{2} + \frac{7}{10} - \frac{2}{5}$

(iv) $7\frac{1}{3}+5\frac{8}{9}-4\frac{1}{27}$ (v) $4\frac{5}{6}-1\frac{4}{5}-\frac{2}{3}$ (vi) $2\frac{2}{3}+1\frac{3}{5}+6\frac{7}{10}-\frac{1}{2}$

(vii) $1\frac{1}{2}+3\frac{3}{4}-7\frac{5}{6}+9\frac{1}{3}$ (viii) $1\frac{7}{12}+8\frac{5}{9}-4\frac{5}{6}-2\frac{1}{18}$

2. Find the following.

(i) $\frac{4}{7}\times\frac{5}{8}$ (ii) $3\frac{1}{4}\times5\frac{1}{3}\times1\frac{7}{13}$ (iii) $\frac{3}{4}$ of $\left(2\frac{3}{7}\times1\frac{15}{34}\right)$ (iv) $1\frac{17}{23}$ of $\left(\frac{6}{5}-\frac{17}{18}\right)$

(v) $\frac{3}{4}$ of 16 m (vi) $\frac{3}{8}$ of $5\frac{1}{3}$ hours (vii) $4\frac{2}{7}$ times ₹35 (viii) $\frac{8}{15}$ of $11\frac{1}{4}$ kg

3. Find the values of the following.

(i) $3\frac{3}{8}\div2\frac{13}{16}$ (ii) $\frac{25}{27}\div\frac{40}{81}$ (iii) $7\frac{1}{3}\div\left(\frac{1}{2}+\frac{2}{5}\right)$ (iv) $12\frac{1}{2}\div\frac{5}{7}$ of $4\frac{2}{3}$

(v) $\frac{2}{3}\times\frac{1}{4}-\frac{1}{12}\div\frac{1}{2}$ (vi) $\left(\frac{2}{3}\times\frac{1}{4}-\frac{1}{12}\right)\div\frac{1}{2}$ (vii) $\frac{2}{3}\times\left(\frac{1}{4}-\frac{1}{12}\div\frac{1}{2}\right)$ (viii) $\frac{2}{3}\times\left(\frac{1}{4}-\frac{1}{12}\right)\div\frac{1}{2}$

4. Simplify the following.

(i) $\dfrac{3\frac{1}{2}}{5\frac{1}{7}}$ (ii) $\dfrac{1\frac{1}{3}+3\frac{1}{4}}{5\frac{2}{7}+\frac{3}{7}}$ (iii) $\dfrac{4\frac{1}{6}-2\frac{1}{3}}{7\frac{1}{8}+6\frac{3}{4}}$ (iv) $\dfrac{2\frac{1}{2}\div7\frac{1}{8}}{4\frac{1}{6}\div3\frac{1}{9}}$

(v) $1\frac{1}{5}\times\left[\left(6\frac{1}{3}-3\frac{2}{5}\right)\div4\frac{2}{5}\right]$ (vi) $4\frac{1}{4}-\left[\frac{1}{2}+\left\{3\frac{1}{5}-\left(\frac{1}{5}-\frac{1}{6}\right)\right\}\right]$

(vii) $7\frac{2}{3}+\left\{2\frac{1}{5}-\left(\frac{2}{3}\times\overline{\frac{3}{4}-\frac{1}{5}}\right)\right\}$ (viii) $2\frac{2}{5}$ of $\left[\frac{5}{8}+\left\{1\frac{4}{7}\div\left(\frac{1}{5}+\frac{3}{7}\right)\right\}\times\frac{3}{8}\right]$

ANSWERS

1. (i) $3\frac{1}{6}$ (ii) $\frac{5}{8}$ (iii) $2\frac{4}{5}$ (iv) $9\frac{5}{27}$ (v) $2\frac{11}{30}$ (vi) $10\frac{7}{15}$ (vii) $6\frac{3}{4}$ (viii) $3\frac{1}{4}$

2. (i) $\frac{5}{14}$ (ii) $26\frac{2}{3}$ (iii) $2\frac{5}{8}$ (iv) $\frac{4}{9}$ (v) 12 m (vi) 2 hours (vii) ₹150 (viii) 6 kg

3. (i) $1\frac{1}{5}$ (ii) $1\frac{7}{8}$ (iii) $8\frac{4}{27}$ (iv) $3\frac{3}{4}$ (v) 0 (vi) $\frac{1}{6}$ (vii) $\frac{1}{18}$ (viii) $\frac{2}{9}$

4. (i) $\frac{49}{72}$ (ii) $\frac{77}{96}$ (iii) $\frac{44}{333}$ (iv) $\frac{224}{855}$ (v) $\frac{4}{5}$ (vi) $\frac{7}{12}$ (vii) $9\frac{1}{2}$ (viii) $3\frac{3}{4}$

Application of Fractions

The knowledge of fractions is often useful in real life. For example, if we know that a boy's pocket money is ₹200 and that he has spent $\frac{1}{4}$ of the money, we can calculate how much he has spent.

Solved Examples

EXAMPLE 1 **Rajesh pays $\frac{3}{4}$ of the cost price of a television in cash. He pays the rest of the amount in 13 monthly instalments of ₹200 each. Find the cost price of the TV.**

Solution The amount paid in cash is $\frac{3}{4}$ of the cost price.

$\therefore$ remaining amount $= \left(1 - \frac{3}{4}\right)$ of the cost price $= \frac{1}{4}$ of the cost price.

Also, the total amount paid in monthly instalments is ₹200 × 13 = ₹2600.

$\therefore$ $\frac{1}{4}$ of the cost price = ₹2600.

$\therefore$ the cost price of the television is ₹2600 $\div \frac{1}{4}$ = ₹2600 × 4 = ₹10 400.

EXAMPLE 2 **A basket had 250 mangoes. $\frac{1}{10}$ were rotten. Rakesh and his friends had $\frac{1}{9}$ of the remaining mangoes. What fraction of the total number of mangoes was left?**

Solution The number of rotten mangoes was $\frac{1}{10}$ of 250 $= \frac{1}{10} \times 250 = 25$.

$\therefore$ number of mangoes left = 250 − 25 = 225.

Rakesh and his friends had $\frac{1}{9}$ of 225 mangoes, i.e., $\frac{1}{9} \times 225 = 25$ mangoes.

$\therefore$ number of mangoes left finally = 225 − 25 = 200.

So, the required fraction is $\frac{200}{250} = \frac{4}{5}$.

EXAMPLE 3 **Chris had 48 peanuts. He gave two thirds of these to squirrels and three fourths of the remaining to crows. How many peanuts were left with him?**

Solution The number of peanuts given to the squirrels was $\frac{2}{3}$ of 48 $= \frac{2}{3} \times 48 = 32$.

$\therefore$ number of remaining peanuts = 48 − 32 = 16.

$\therefore$ number of peanuts given to the crows $= \frac{3}{4}$ of 16 $= \frac{3}{4} \times 16 = 12$.

So, the number of peanuts left finally was 16 − 12 = 4.

EXERCISE 2C

1. The cost price of a microwave oven is ₹12 600. Rajesh paid $\frac{2}{3}$ of the price in cash and the rest in 12 equal monthly instalments. Find the amount of the monthly instalment.
2. Aftab's monthly salary is ₹25 000. He spends $\frac{1}{5}$ of his salary on rent. Of the remaining amount, he spends $\frac{1}{4}$ on the education of his children and $\frac{1}{5}$ on food. Find (i) Aftab's monthly expenditure and (ii) his monthly savings.

3. A piece of wire $\frac{3}{4}$ m long was cut into three pieces. The first piece was $\frac{3}{20}$ m long, while the length of the second piece was $\frac{5}{3}$ of the first. How long was the third piece?

4. John got a prize of ₹75 000 in a TV contest. He paid $\frac{1}{10}$ of the prize as the income tax. Out of the remaining, he gave $\frac{1}{3}$ to his son, $\frac{1}{5}$ to his daughter, $\frac{1}{10}$ to the Red Cross Society, and the rest to his wife. What portion of the prize did John's wife get and how much?

5. A shopkeeper has a stock of 140 kg of wheat. In the first four weeks, he sells $20\frac{1}{2}$ kg, $25\frac{3}{4}$ kg, $26\frac{1}{2}$ kg and $47\frac{1}{4}$ kg of wheat respectively. How much wheat and what portion of the stock does he sell?

6. Rajesh had a packet of 20 sketch pens. He gave 12 to Krishna and 6 to Meena. What fractions of the packet did he give to Krishna and Meena? What fraction of the packet was left with him?

7. Ajit and Kunal paid $\frac{5}{7}$ and $\frac{2}{7}$ of the total cost of a book of comics. If the book cost ₹84, how much did each of them spend?

8. Rakesh bought $1\frac{1}{4}$ kg of grapes, $2\frac{1}{2}$ kg of mangoes, $1\frac{1}{2}$ kg of apples and 2 kg of oranges. How much fruits in total did he buy?

9. A book has 360 pages. Ajay reads $\frac{11}{18}$ of the book in a week, while Seema reads $\frac{7}{10}$ of the book in a week. Who reads faster, by what fraction and by how many pages?

10. Lalit weighs 66 kg. Gauri's weight is $\frac{4}{3}$ of Lalit's weight. Find (i) Gauri's weight and (ii) the difference between their weights.

ANSWERS

1. ₹350	2. (i) ₹14 000 (ii) ₹11 000
3. $\frac{7}{20}$ m	4. $\frac{33}{100}$ and ₹24 750
5. 120 kg and $\frac{6}{7}$ respectively	6. Krishna: $\frac{3}{5}$ and Meena: $\frac{3}{10}$; $\frac{1}{10}$
7. ₹60 and ₹24 respectively	8. $7\frac{1}{4}$ kg
9. Seema: $\frac{4}{45}$ and 32 respectively	10. (i) 88 kg (ii) 22 kg

❖

Revision Exercise 1

1. Find
(i) $(-21) \times (-30) \times (-5) \div 7 \times (-20)$ (ii) $(-141) \div 9 \times (-189) \div (-21)$ (iii) $256 \div [(-25) - (-9)]$

2. Simplify the following.
(i) $72 \div 6 + 3 \times 2$ (ii) $72 \div (6 + 3) \times 2$ (iii) $72 \div (6 + 3 \times 2)$

3. Find the following.
(i) $48 \div 8 + 4 \div 2$ (ii) $42 \div 7 + 14 - 4 \times 5$ (iii) $46 + 12 \div 3 \times 4 + 2$

4. Simplify $480 \div 2$ of $[40 - \{21 + (36 \div 6 + 3)\}]$.

5. In a class test, four marks are awarded for every correct answer, one mark is deducted for every wrong answer, and zero marks are awarded for unattempted questions. Riya attempted twenty questions. Out of her answers, five were wrong. Find the marks obtained by Riya.

6. Find the following.
(i) $1\frac{2}{5}$ of $4\frac{2}{7}$ km (ii) $\frac{2}{9}$ of $7\frac{1}{5}$ hours (iii) $1\frac{4}{11}$ times ₹363

7. Find the values of the following.
(i) $1\frac{1}{2} + \left(3\frac{1}{3} + 4\frac{1}{5} - 6\frac{1}{2}\right)$ (ii) $1\frac{7}{53}$ of $\left[1\frac{1}{5} - \left\{3\frac{4}{5} + \left(\frac{1}{2} + \frac{1}{3}\right)\right\} \times 3\frac{3}{4}\right]$

(iii) $1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2}}}$

8. Out of the fractions $\frac{5}{7}, \frac{4}{9}, \frac{6}{11}, \frac{2}{5}$ and $\frac{3}{4}$, find the difference between the largest fraction and the smallest fraction.

9. The monthly income of Tom is ₹120 000. He spends $\frac{1}{6}$ of his income on the education of his children, and $\frac{1}{3}$ on the repayment of his housing-loan instalment. Of the remaining amount, he spends $\frac{1}{4}$ on food and $\frac{2}{5}$ on the rest items. Find the sum of money left with him.

ANSWERS

1. (i) 4200 (ii) −117 (iii) −16
2. (i) 18 (ii) 16 (iii) 6
3. (i) 8 (ii) 0 (iii) 64
4. 16
5. 55
6. (i) 6 km (ii) $1\frac{3}{5}$ hours (iii) ₹495
7. (i) $1\frac{14}{31}$ (ii) −18 (iii) $\frac{8}{5}$
8. $\frac{11}{35}$
9. ₹21 000